**King Saud University**

**Department of Mathematics**

**Second Mid Term Exam**

ACTU 361 - Mathematics of Finance (1)

(2/7/1437 H, Time 1H30)

**Exercise 1. [8]**

We consider the following annuity:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Pay. | 1 | 1+k | $$(1+k)^{2}$$ | … | $$(1+k)^{n-1}$$ | 0 |
| Time | 0 | 1 | 2 | … | n-1 | n |

1. Prove that the present value of this annuity for an effective annual interest rate$ i $ is equal to:

$$(G\ddot{a})\_{\overbar{n/}i}=\ddot{a}\_{\overbar{n/}\frac{i-k}{1+k}}$$

1. Deduce the present value $(Ga)\_{\overbar{n/}i,r} $of the following annuity

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Pay. | 1 | 1+k | $$(1+k)^{2}$$ | … | $$(1+k)^{n-1}$$ | $$(1+k)^{n}$$ |
| Time | 1 | 2 | 3 | … | n-1 | n |

1. A 10 year annuity pays 100 at the beginning of year 1, with subsequent payments increase in such a way that each payment is 4% greater than the previous one. Find the present value of this annuity if the annual effective rate of interest is 5%.

**Exercise 2. [5]**

We consider an annuity that pays 55 at the end of year 1, 50 at the end of year 2, 45 at the end of year 3 and so on until the last payment is 10.

Find the present value and the accumulated value of this annuity if the rate of interest is 2%.

**Exercise 3. [8]**

You are giving the following information about a loan of L that is to be repaid with a series of 25 annual payments at the end of each year:

1. The first 6 payments are 1000 each one and are due 3 years from now.
2. The next 9 payments are each 3 % larger than the preceding payment.
3. From the 16th to the 25th payment, each payment will be 50 greater than the preceding payment.
4. The annual effective interest rate is 4%.
5. Find the present value of the first 5 payments.
6. Find the present value of the 10 intermediary payments
7. Find the present value of the last 10 payments
8. Deduce L
9. **Exercise 4. [4]**

Determine the present value of an annuity that pays a rate of $3(10-t) $ at time t for 10 years with a continuous compounding interest rate of 6%.