King Saud University	College of Sciences	Department of Mathematics
Midterm Examination	Math 228	1444(I) Time:2H

Name: Answer .... Student Number: .M.e...... Grade: 4/30

15 **Question 1:** Evaluate the following integrals:

1. $\int x^2 e^{-x}\,dx$. (3 marks)

$u = x^2 \longrightarrow du = 2x\,dx$.

$d\vartheta = e^{-x}dx \longrightarrow \vartheta = -e^{-x}$ 1

$\int x^2 e^{-x}dx = -x^2e^{-x} - \int 2x(-e^{-x})\,dx$.

$= -x^2e^{-x} + 2\int xe^{-x}dx$. 1

$u = x \longrightarrow du = dx$.

$d\vartheta = e^{-x}dx \longrightarrow \vartheta = -e^{-x}$.

$\int x^2e^{-x}dx = -x^2e^{-x} + 2xe^{-x} + 2\int e^{-x}dx$.

$= -x^2e^{-x} + 2xe^{-x} - 2e^{-x} + c$. 1

2. $\int \cos^3 x \sin^4 x\,dx$. (3 marks)

$\int \cos^3 x \sin^4 x\,dx = \int \cos^2 x \sin^4 x \cos x\,dx$

$= \int (1-\sin^2 x)\sin^4 x \cos x\,dx$. 1 $u = \sin x$, $du = \cos x\,dx$ 1

$= \int (1-u^2)u^4\,du$

$= \int (u^4 - u^6)\,du$

$= \frac{u^5}{5} - \frac{u^7}{7} + c$

$= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$. 1

3. $\int \cos(5x)\cos(3x)\,dx$. (3 marks)

$$\int \cos 5x \cos 3x \,dx = \int \frac{1}{2}\left[\cos(5x+3x) + \cos(5x-3x)\right] dx$$

$$= \frac{1}{2}\int (\cos 8x + \cos 2x)\,dx \quad (1)$$

$$= \frac{1}{2}\int \cos 8x \,dx + \int \cos 2x \,dx \quad (1)$$

$$= \frac{1}{2}\left[\frac{\sin 8x}{8} + \frac{\sin 2x}{2}\right] + C$$

$$= \frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C. \quad (1)$$

4. $\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$ (3 marks)

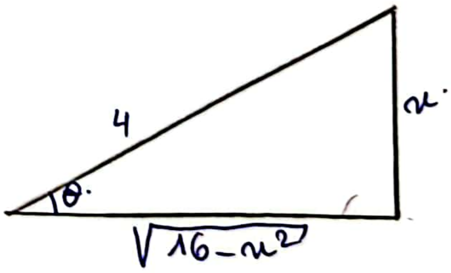
$x = 4\sin\theta$; $dx = 4\cos\theta\,d\theta$. (1)

$$\int \frac{1}{x^2\sqrt{16-x^2}}\,dx = \int \frac{1}{16\sin^2\theta\sqrt{16-16\sin^2\theta}}\,4\cos\theta\,d\theta$$

$$= \frac{1}{16}\int \frac{1}{\sin^2\theta\,\cancel{4}\sqrt{1-\sin^2\theta}}\,\cancel{4}\cos\theta\,d\theta.$$

$$= \frac{1}{16}\int \frac{1}{\sin^2\theta}\,\frac{\cos\theta}{\cos\theta}\,d\theta$$

$$= \frac{1}{16}\int \csc^2\theta\,d\theta = -\frac{1}{16}\cot\theta + C. \quad (1)$$



$$\Rightarrow \cot\theta = \frac{\sqrt{16-x^2}}{x}.$$

$$= -\frac{1}{16}\frac{\sqrt{16-x^2}}{x} + C. \quad (1)$$

5. $\int \frac{4x^2+13x-9}{x^3+2x^2-3x}\,dx$ (3 marks)

$x^3+2x^2-3x = x(x^2+2x-3) = x(x+3)(x-1)$.

$\frac{4x^2+13x-9}{x^3+2x^2-3x} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-1} = \frac{A(x+3)(x-1)+B(x-1)x+Cx(x+3)}{x^3+2x^2+3x}$.

$\Rightarrow 4x^2+13x-9 = A(x+3)(x-1)+B(x-1)x+Cx(x+3)$.

$x=0 \Rightarrow -9 = A(-3) \Rightarrow A=3$,

$x=-3 \Rightarrow -12 = B(+12) \Rightarrow B=-1$. 1,5

$x=1 \Rightarrow 8 = C(4) \Rightarrow C=2$.

$\Rightarrow \int \frac{4x^2+13x-9}{x^3+2x^2-3x}\,dx = \int \frac{3}{x}\,dx - \int \frac{1}{x+3}\,dx + 2\int \frac{1}{x-1}\,dx$ 0,5

$= 3\ln|x| - \ln|x+3| + 2\ln|x-1| + C$. 1

6 **Question 2:** Find the extremum and saddle points of the following functions:

1. $f(x,y) = -x^2-y^2-4x+2y-1$. (3 marks)

$\frac{\partial f}{\partial x} = -2x-4$, $\frac{\partial f}{\partial y}(x,y) = -2y+2$,

1,5

$\begin{cases} -2x-4=0 \\ -2y+2=0 \end{cases} \Rightarrow x=-2;\ y=1;\ \Rightarrow (-2,1)$ critical point.

$\frac{\partial^2 f}{\partial x^2} = -2$; $\frac{\partial^2 f}{\partial y^2} = -2$; $\frac{\partial^2 f}{\partial x \partial y} = 0$.

0,5

$\Rightarrow D(x,y) = 4$

$D(-2,1) = 4 > 0$,

$\frac{\partial^2 f}{\partial x^2}(-2,1) = -2 < 0 \Rightarrow f(-2,1)$ is a local maximum for f.

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2. $g(x,y) = x^3 + 3xy - y^3$. (3 marks)

$\frac{\partial f}{\partial x}(x,y) = 3x^2 + 3y$, $\frac{\partial f}{\partial y}(x,y) = 3x - 3y^2$.

$\begin{cases} 3x^2 + 3y = 0 \\ 3x - 3y^2 = 0 \end{cases} \Rightarrow \begin{cases} x^2 + y = 0 \\ x = y^2 \end{cases} \Rightarrow \begin{cases} y^4 + y = 0 \\ x = y^2 \end{cases} \Rightarrow \begin{cases} y(y^3+1) = 0 \\ x = y^2 \end{cases}$

1,5

$\Rightarrow \begin{cases} y = 0 \\ x = 0 \end{cases}$ or $\begin{cases} y = -1 \\ x = 1 \end{cases} \Rightarrow (0,0), (1,-1)$ are critical points of f.

$\frac{\partial^2 f}{\partial x^2}(x,y) = 6x$; $\frac{\partial^2 f}{\partial y^2}(x,y) = -6y$; 0,5 $\frac{\partial^2 f}{\partial x \partial y}(x,y) = 3$.

$\Rightarrow D(x,y) = -36xy - 3$.

$D(0,0) = -3 < 0$ (0,0) saddle point. 0,5

$D(1,-1) = 36 - 3 = 33 > 0$.

$\frac{\partial^2 f}{\partial x^2}(1,-1) = 6 > 0 \Rightarrow f(-1,1)$ is a local ~~maximum~~ minimum for f.

0,5

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Question 3:

1. Evaluate $\int_1^4 \int_{-1}^2 (2x + 6x^2 y)\, dy\, dx$. (3 marks)

$\int_1^4 \int_{-1}^2 (2x + 6x^2 y)\, dy\, dx$

$= \int_1^4 \left[2xy + 3x^2y^2\right]_{-1}^{2} dx$ 1

$= \int_1^4 \left[(4x + 12x^2) - (-2x + 3x^2)\right] dx$

$= \int_1^4 (6x + 9x^2)\, dx$ 1

$= \left[3x^2 + 3x^3\right]_1^4 = 234$. 1

2. Evaluate $\int_1^3 \int_{\frac{\pi}{6}}^{y^2} 2y \cos x \, dx \, dy$. (3 marks)

$$\int_1^3 \int_{\frac{\pi}{6}}^{y^2} (2y \cos x) \, dx \, dy.$$

$$= \int_1^3 \left[2y \sin x \right]_{\frac{\pi}{6}}^{y^2} dy \quad 1$$

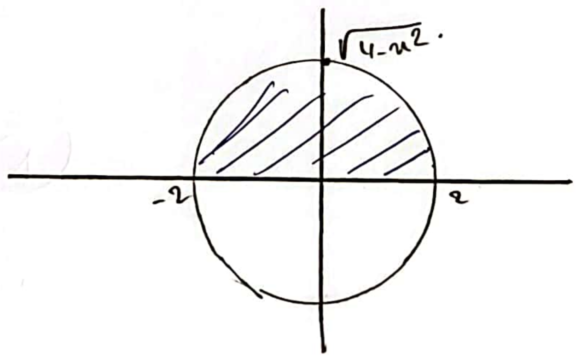
$$= \int_1^3 (2y \sin y^2 - y) \, dy \quad 1$$

$$= \left[-\cos y^2 - \frac{y^2}{2} \right]_1^3 = \left(-\cos 9 - \frac{9}{2}\right) - \left(-\cos 1 - \frac{1}{2}\right).$$

$$= -\cos 9 + \cos 1 - 4. \quad 1$$

3. Use polar coordinates to evaluate $\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} (x^2 + y^2)^{\frac{3}{2}} \, dy \, dx$. (3 marks)

$0 \leq \theta \leq \pi$; $0 \leq r \leq 2$; 1



$$\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} (x^2 + y^2)^{3/2} \, dy \, dx$$

$$= \int_0^{\pi} \int_0^{2} r^3 \, r \, dr \, d\theta \quad 1$$

$$= \int_0^{\pi} d\theta \int_0^2 r^4 \, dr$$

$$= \pi \cdot \left[\frac{r^5}{5} \right]_0^2 = \frac{32}{5}\pi \quad 1.$$