King Saud University
Department of Mathematics

Final Exam

1^{st} **semester 1437 H**

Course Title: Math 244 (Linear Algebra)

Date: Tuesday 8 December 2015; (3-4:45) pm

(. . . . . .) Name . . . . ID . . . . Section . . . .

Lecturer . . . .

Question	Grade
Q1	
Q2	
Q3	
Q4	
Total	

Part I	(a)	(b)	(c)	(d)	(e)	(f)
Answer						

Question 1

I. Choose the correct answer (write it down on the table above):

(a) (2) If $u = (2, \frac{1}{2}, 1, 1)$, $v = (0, 2, 1, -2)$ and $w = (1, 4, 2, -2)$ then $\|-2u\| + \|v\|$ and $\left\| \frac{1}{\|w\|} w \right\|$ are respectively

(i) -4 and $3\sqrt{2}$
(ii) -4 and 1
(iii) 8 and 1
(iv) None.

(b) The angle θ between $u = (-4, 0, 2, -2)$ and $v = (2, 0, -1, 1)$ is

(i) 0
(ii) $\frac{\pi}{2}$
(iii) π
(iv) None.

(c) If A, B and C are 2×2 matrices with $\det(A) = 2$, $\det(B) = -1$ and $\det(C) = 5$ then $\det(A^{-1}B^TC^2)$ equals

(i) $-\frac{25}{2}$
(ii) 50
(iii) -5
(iv) None

(d) The solution space of $\begin{bmatrix} 1 & -2 & 6 \\ 3 & -6 & 18 \\ -7 & 14 & -42 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(i) A line through the origin
(ii) The origin
(iii) A plane through the origin
(iv) None

(e) The cofactor C_{33} of the matrix $A = \begin{bmatrix} 3 & 3 & 5 & 1 \\ 2 & 2 & 1 & 0 \\ 0 & 2 & 4 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ is

(i) -2
(ii) 2
(iii) 0
(iv) None

(f) If $\vec{u} = (2, -2)$; $\vec{v} = (5, 8)$ and $\vec{w} = (-4, 3)$, then $(\vec{u} \cdot \vec{v})\vec{w}$ equals

(i) $(-24, 18)$
(ii) 6
(iii) $(24, -18)$
(iv) None

II. Determine wether the following is true or false.

(a) If $\vec{u}$ and $\vec{v}$ are orthogonal vectors then for all nonzero scalars k_1 and k_2, $k_1\vec{u}$ and $k_2\vec{v}$ are orthogonal vectors. [ ]

(b) If $C = [c_{ij}]_{3\times3}$ and D is obtained fro C by adding 3 times the second row to each of the first and third rows, then $\det(D) = 9\det(C)$ [ ]

(c) The set $W = \{(x_1, x_2) \in R^2 : x_1 \geq 0\text{and } x_2 \geq 0\}$ with the standard operations is a subspace of R^2 [ ]

(d) $S = \{(-2, 0, 1), (4, -2, 0), (6, -6, 3)\}$ is linearly dependent set in R^3 [ ]

(e) $S = \{(2, -1, 2), (4, 1, 3), (2, 2, 1)\}$ spans R^3 [ ]

(f) The subspaces of R^2 are $\{\vec{0}\}$ and R^2 itself [ ]

Question 2

(i) Determine if the set $W = \{(x, y) : x, y \in R\}$ with the operations

$$(x, y) + (x', y') = (xx', yy')$$

and

$$k(x, y) = (kx, ky)$$

form a vector space or not. Justify your answer.

(ii) Determine the value of a such that the matrices

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -4 \\ a & -2 \end{bmatrix}$$

are linearly independent

Question 3

(a) Determine all the vectors $\vec{v} = (v_1, v_2)$ in R^2 that are orthogonal to $\vec{u} = (4, 2)$

(b) Prove that $W = \{A \in M_{2\times2}(R) : A = A^T\}$ is a subspace of $M_{2\times2}(R)$.

(c) Given $\vec{u} \cdot \vec{u} = 39$, $\vec{u} \cdot \vec{v} = -3$ and $\vec{v} \cdot \vec{v} = 79$. Find $(\vec{u} + 2\vec{v}) \cdot (3\vec{u} + \vec{v})$.

Question 4

(a) Determine the determinant of

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$$

.

(b) Prove that for any $\vec{u}, \vec{v} \in V$,

$$\|\vec{u} + \vec{v}\|^2 + \|\vec{u} - \vec{v}\|^2 = 2\|\vec{u}\|^2 + 2\|\vec{v}\|^2.$$

(c) Use the adjoint of $A = \begin{bmatrix} 1 & -5 & -1 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ to find A^{-1}.