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١٣٧٧-١٤٢٧هـ

STAT 324
Second Midterm Exam
First Semester
1431 – 1432 H

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	اسم الدكتور		رقم الشعبة

- Mobile Telephones are <u>not allowed</u> in the classrooms.
- Time allowed is <u>90 minutes</u>.
- Answer all questions.
- Choose the nearest number to your answer.
- WARNING: Do not copy answers from your neighbors. <u>They have different questions forms.</u>
- For each question, put the code of the correct answer in the following table beneath the question number:

1	2	3	4	5	6	7	8	9	10
d	c	a	d	b	a	c	c	c	d

11	12	13	14	15	16	17	18	19	20
a	b	b	d	a	b	a	b	b	c

21	22	23	24	25	26	27	28	29	30
c	c	b	a	a			a	b	c

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Assume that the mean life of a machine is 6 years with a standard deviation of 1 year. Suppose that the life of such machines follows approximately a normal distribution. If a random sample of 4 is selected from these machines, then:

1-The sample mean $\overline{X}$ has a standard error is:

(a) 0.70 (b) 0.79 (c) 0.25 (d) 0.50

2- $P(\overline{X} < 7.2)$ is

(a) 0.5212 (b) 0.8505 (c) 0.9918 (d) 0.7881

3- $P(\overline{X} > k) = 0.1492,$ then the value of k is :

(a) 6.52 (b) 0.8505 (c) 0.1492 (d)6.73

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4- If the standard error of the mean for the sampling distribution of random samples of size 36 from a large population is 2, how large must the size of the sample become if the standard error is to be reduced to 1.2?

(a) 36 (b) 120 (c) 22 (d) 100

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For a t distribution find

5- $t_{0.025}$ with 14 degrees of freedom

(a) 2.160 (b) 2.145 (c) 2.131 (d) 2.120

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The heights of a random sample of 50 college students showed the mean $\overline{X} = 174.5$ centimeters and suppose that the standard deviation of the heights is 6.9 centimeters.

6-The lower bound of 98 % confidence interval for the mean height of all college students is:

(a) 172.23 (b) 174.50 (c) 176.77 (d) 167.60

7- The maximum size of error if we estimate the mean height (μ) by $\overline{X}$ with 98 % confidence degree, is:

(a) 5.15 (b) 2.33 (c) 2.27 (d) 2.58

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A random sample of size 25 is taken from a normal population having a mean of 80 and a standard deviation of 5. A second independent random sample of size 36 is taken from a different normal population having a mean of 75 and a standard deviation of 3. Then,

8-P($\overline{x}_1 - \overline{x}_2 \leq 2$) =

(a) 0.0009 (b) 174.50 (c) 0.0037 (d) 0.2154

9-P($1.5 \leq \overline{x}_1 - \overline{x}_2 \leq 2$) =

(a) 0.1254 (b) 0.3451 (c) 0.0028 (d) 0.0009

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On past evidence, an electrical component has a probability of 0.8 of being satisfactory (not defective). In a sample size of five components chosen randomly,

10-The probability of getting one defective is:

(a) 0.200 (b) 0.0064 (c) 0.8 (d) 0.4096

11-The probability of getting two or more defectives is:

(a) 0.2627 (b) 0.7373 (c) 0.2048 (d) 0.0512

12-The expected number of defectives in the sample is:

(a) 4.0 (b) 1.0 (c) 0.8 (d) 0.2

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The number of calls that arrive at a switchboard during one hour is poisson distributed with mean 5 calls.

13-The probability of exactly two calls arrive during an hour is:

(a) 0.423 (b) 0.0842 (c) 0.9158 (d) 0.1246

14-The probability of at most two calls arrive during an hour is:

(a) 0.875 (b) 0.084 (c) 0.0404 (d) 0.1247

15-The probability of exactly two calls arrive during two hours is:

(a) 0.0023 (b) 0.8754 (c) 0.8 (d) 0.1246

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An office has 10 employees, three men and seven women. The manager chooses four at random to attend a short course on quality improvement.

16-The probability that an equal number of men and women are chosen is:

(a) 0.5 (b) 0.3 (c) 0.7 (d) 0.21

17-The probability that more women are chosen is:

(a) 0.6667 (b) 0.500 (c) 0.1667 (d) 0.3333

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If X is a continuous uniform random variable in the interval (-1 , 1).

18-P(X>0.5) equal:

(a) 0.51 (b) 0.25 (c) 0.00 (d) 0.75

19-The expected value of X , E(X) equal:

(a) 0.5 (b) 0.0 (c) 0.33 (d) 0.25

20-The variance of X , Var(X) equal:

(a) 0.5 (b) 0.0 (c) 0.33 (d) 0.25

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Assume that 15 % of the students in a certain university smoke cigarettes. A random sample of 35 students is taken from this university. If $\hat{P}$ is the proportion of smokers in the sample, then:

21-The expected value of $\hat{P}$ is:

(a) 0.85 (b) 0.80 (c) 0.15 (d) 0.35

22- $P(\hat{P} > 0.17)$ is:

(a) 0.0094 (b) 0.0166 (c) 0. 3707 (d) 0.8515

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Suppose that 7 % of the pieces from a production process A are defective while that proportion for another production process B is 5 %. A random sample of size 400 pieces is taken from the production process A while the sample size taken from the production process B is 300 pieces. If $\hat{P}_1$ and $\hat{P}_2$ be the proportions of defective pieces in the two samples, respectively, then:

23-The sampling distribution of $\hat{P}_1 - \hat{P}_2$ is:

(a) Standard normal (b) normal (c) T (d) Unknown,

24- The standard error of $(\hat{P}_1 - \hat{P}_2)$ is:

(A) 0.0179 (B) 0.10 (C) 0 (D) 0.22

25- $P(\hat{P}_1 - \hat{P}_2 > 0)$ is:

(a) 0.8686 (b) 0.1314 (c) 0 (d) 1

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The lifetime of a certain electronic component is known to be exponentially distributed with mean life of 100 hours.

26-The probability that a component will fail between 70 and 90 hours is:

(a) 0.496 (b) 0.406 (c) 0.09 (d) 0.222

27-The proportion of such component will fail before 50 hours is:

(a) 0.5 (b) 39.35% (c) 50% (d) 0.4066

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The length of a batch of steel rods are normally distributed with mean 10 ft and standard deviation 2 ft.

28-The proportion of rods which are longer than 13 ft is:

(a) 6.68% (b) 1.5% (c) 98.5% (d) 0.15

29-If P(X>K1)=0.05 , then the value of K1 is:

(a) 11.6 (b) 13.3 (c) 6.7 (d) -6.72

30-The value of P(X>10) is:

(a) 1.5 (b) 1.0 (c) 0.5 (d) 0.0

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Midterm 2
1431-1432

$X \sim N(\mu, \sigma)$

$\mu = 6 \quad , \sigma = L \qquad \boxed{n = 4}$

1- $s.e.(\bar{X}) = \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{4}} = 0.50$

2- $\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$

$P(\bar{X} < 7.2) = P(Z < \frac{7.2 - 6}{0.5}) = P(Z < 2.4)$
$= 0.9918$

3- $P(\bar{X} > k) = 0.1492$

$P(\bar{X} \leq k) = 1 - 0.1492$

$P(Z < \frac{k - 6}{0.5}) = 0.8508$

$\frac{k - 6}{0.5} = 1.04 \quad , \quad \underline{k = 6.52}$

4- $s.e.(\bar{X}) = \sigma_{\bar{X}} = 2 \quad , n = 36$

$\therefore \frac{\sigma}{\sqrt{36}} = 2 \quad \Rightarrow \quad \underline{\sigma = 12}$

$\frac{12}{\sqrt{n}} = 1.2 \quad \Rightarrow \quad \sqrt{n} = 10 \quad \Rightarrow \quad \underline{n = 100}$

5- $t_{0.025} \quad , \nu = 14 \quad \Rightarrow \quad t = 2.145$

6- $n = 50 \quad , \quad \bar{X} = 174.5 \quad , \sigma = 6.9$

$\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 174.5 - 2.32 * \frac{6.9}{\sqrt{50}} = \boxed{172.23}$

98% $\quad (1 - \alpha) = 0.98 \Rightarrow \alpha = 0.02 \Rightarrow \frac{\alpha}{2} = 0.01$

$P(Z > Z_{\frac{\alpha}{2}}) = 0.01$

$P(Z < Z_{\frac{\alpha}{2}}) = 0.99 \quad \Rightarrow \quad \frac{\alpha}{2} = 2.32$

7- $e = Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 2.32 * \frac{6.9}{\sqrt{50}} = \underline{2.27}$

8- $n_1 = 25 \quad , \mu_1 = 80 \quad , \sigma_1 = 5 \qquad \bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}})$

$n_2 = 36 \quad , \mu_2 = 75 \quad , \sigma_2 = 3 \qquad \sim N(5, 1.118)$

- $P(\bar{X}_1 - \bar{X}_2 \leq 2) = P(Z \leq \frac{2 - 5}{1.118}) = P(Z \leq -2.68) = 0.0037$

9- • $P(1.5 \leq \bar{X}_1 - \bar{X}_2 \leq 2) = P(\frac{1.5 - 5}{1.118} \leq Z \leq \frac{2 - 5}{1.118}) = P(Z \leq -2.68) - P(Z \leq -3.13)$
$= 0.0037 - 0.0009$

$X \sim \text{binomial}(n, p) \qquad n = 5 \;,\; p = 0.8$

10- prob. of defective $p = 0.2 \;,\; q = 0.8$

X ... no. of defectives in sample

$$P(X=1) = \binom{5}{1}(0.2)^1(0.8)^4 = 0.4096$$

11-
$$\begin{aligned} P(X \geq 2) &= 1 - P(X<2) \\ &= 1 - [P(X=0) + P(X=1)] \\ &= 1 - \left[\binom{5}{0}(0.8)^5 + 0.4096\right] = 0.26272 \end{aligned}$$

12- $E(X) = np = 5 * (0.2) = 1$

$$X \sim \text{Poisson}(\mu) \qquad f(x) = e^{-\mu}\frac{\mu^x}{x!} \;,\; x = 0,1,2,\ldots$$

13- $\mu = 5 * 1 = 5 \qquad P(X=2) = e^{-5} \cdot \dfrac{5^2}{2!} = 0.0842$

14- $\mu = 5*1 = 5$
$$\begin{aligned} P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\ &= e^{-5}\left[\frac{5^0}{0!} + \frac{5}{1!} + \frac{5^2}{2!}\right] = 0.1247 \end{aligned}$$

15- $\mu = 5 * 2 = 10 \qquad P(X=2) = e^{-10} \cdot \dfrac{10^2}{2!} = 0.0023$

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X ... no. of women chosen

16- $P(X=2) = \dfrac{\binom{7}{2}\binom{3}{2}}{\binom{10}{4}} = 0.3$

17-
$$\begin{aligned} P(X > 2) &= P(X=3) + P(X=4) \\ &= \frac{\binom{7}{3}\binom{3}{1} + \binom{7}{4}\binom{3}{0}}{\binom{10}{4}} = 0.6667 \end{aligned}$$

18-
$$X \sim U(-1, 1) \qquad f(x) = \frac{1}{b-a} \;,\; a \leq x \leq b$$

$$\begin{aligned} P(X > 0.5) &= 1 - P(X \leq 0.5) \\ &= 1 - \int_{-1}^{0.5} \tfrac{1}{2}\, dx = 1 - \tfrac{1}{2}[x]_{-1}^{0.5} = 0.25 \end{aligned}$$

or $P(X > 0.5) = \displaystyle\int_{0.5}^{1} \tfrac{1}{2}\, dx = \tfrac{1}{2}[1 - 0.5] = 0.25$

19- $E(X) = \dfrac{a+b}{2} = 0 \qquad$ 20- $Var(X) = \dfrac{(b-a)^2}{12} = 0.33$

$n = 35$, $P = 0.15$ $q = 0.85$

$\hat{P} \sim N(P, \sqrt{\frac{Pq}{n}})$

21- $E(\hat{P}) = P = 0.15$

22 - $P(\hat{P} > 0.17) = 1 - P(\hat{P} \leq 0.17)$

$= 1 - P(\hat{Z} \leq \frac{0.17 - 0.15}{\sqrt{(0.15)(0.85)/35}})$

$= 1 - P(\hat{Z} \leq 0.33) = 1 - 0.6293 = 0.3707$

$n_1 = 400$ $n_2 = 300$

$P_1 = 0.07$ $P_2 = 0.05$

23 - for large n_1, n_2

$\hat{P}_1 - \hat{P}_2 \sim \text{Normal}(P_1 - P_2, \sqrt{\frac{P_1 q_1}{n_1} + \frac{P_2 q_2}{n_2}})$

$E(\hat{P}_1 - \hat{P}_2) = 0.02$

24- $s.e.(\hat{P}_1 - \hat{P}_2) = \sqrt{\frac{(0.07)(0.93)}{400} + \frac{(0.05)(0.95)}{300}} = 0.0179$

25 - $P(\hat{P}_1 - \hat{P}_2 > 0) = P(\hat{Z} > \frac{0 - 0.02}{0.0179})$

$= P(\hat{Z} > -1.12) = 1 - P(\hat{Z} < -1.12)$

$= 1 - 0.1314 = 0.8686$

$X \sim Exp(\beta)$ $\beta = 100$

$f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}$ $x > 0$

26- $P(70 \leq X \leq 90) = \int_{70}^{90} \frac{1}{\beta} e^{-\frac{x}{\beta}} dx$

$= -e^{-\frac{x}{\beta}} \Big|_{70}^{90} = -e^{-0.9} + e^{-0.7}$

27- $P(X < 50) = \int_0^{50} f(x)\, dx = -e^{-\frac{x}{\beta}} \Big|_0^{50} = 1 - e^{-0.5}$

$X \sim N(\mu, \sigma)$ $\mu = 10$, $\sigma = 2$

28 - $P(X > 13) = 1 - P(Z \leq \frac{13-10}{2}) = 1 - P(Z \leq 1.5)$

$= 1 - 0.9332 = 0.0668$

29 - $P(X > K_1) = 0.05$ $1 - P(Z \leq \frac{K_1 - 10}{2}) = 0.05$

$P(Z \leq \frac{K_1 - 10}{2}) = 0.95$ $\underline{K_1 = 13.3}$

30 - $P(X > 10) = 1 - P(Z \leq 0) = 0.5$