١٣٧٧-١٤٢٧هـ

Department of Statistics
& Operations Research
College of Science,
King Saud University

STAT 324
Second Midterm Exam
Second Semester
1431 – 1432 H

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- Mobile Telephones are not allowed in the classrooms.
- Time allowed is **90 minutes**. (7 - 8.30 PM)
- Answer all questions.
- Choose the nearest number to your answer.
- WARNING: Do not copy answers from your neighbors. They have different questions forms.
- For each question, put the code of the correct answer in the following table beneath the question number. Do not use pencil. Use Capital letters.

1	2	3	4	5	6	7	8	9	10
D	B	E	D	A	D	A	C	B	A

11	12	13	14	15	16	17	18	19	20
A	D	B	C	B	D	D	D	C	A

21	22	23	24	25	26	27	28	29	30
B	C	C	C	A	C	B	A	D	D

⟩⟩⟩⟩ Assume that the length (in minutes) of a particular type of a telephone conversation is a random variable X with a probability density function of the form:

$$f(x) = \begin{cases} \frac{1}{5} e^{-\frac{1}{5}x} & ; x > 0 \\ 0 & ; \text{elsewhere} \end{cases}$$

(1) The average of X is:

(A) 1	(B) 2.5	(C) 0.2	(D) 5	(E) none of these

(2) The standard deviation of X is:

(A) 0.2	(B) 5	(C) 25	(D) 1	(E) none of these

(3) The probability $P(\mu - 2\sigma < X < \mu + 2\sigma)$ will have an exact value equal to:

(A) 0. 9502	(B) 0.2	(C) 0.3181	(D) 0.75	(E) none of these

(4) According to Chebychev theorem the lower bound for the probability $P(\mu - 2\sigma < X < \mu + 2\sigma)$ is:

(A) 0.25	(B) 0.2	(C) 0.0498	(D) 0.75	(E) none of these

⟩⟩⟩⟩ Suppose the weight of a large number of fat persons is normally distributed with mean of 128 kg and a standard deviation of 9 kg.

(5) If we select a person at random, the probability that his weight will be at least 110 kg is:

(A) 0.9772	(B) 0.9982	(C) 0.4207	(D) 0.0228	(E) none of these

(6) The percentage of fat persons with weights less than 149 kg is:

(A) 90.03 %	(B) 1 %	(C) 50 %	(D) 99.01 %	(E) none of these

(7) The weight in kg above which 50 % of those persons will be, is:

(A) 128	(B) 0	(C) 118.28	(D) 119	(E) none of these

⟩⟩⟩⟩ If the random variable X has a normal distribution with the mean μ and the variance σ^2,

(8) Then $P(X < \mu + 2\sigma)$ equals to

(A) 0.0228	(B) 0.6772	(C) 0.9772	(D) 0.2020	(E) none of these

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⟩⟩⟩⟩ If the random variable X has a normal distribution with the mean μ and the variance 1, and if $P(X < 3) = 0.877$,

(9) Then μ equals:

(A) 1.16	(B) 1.84	(C) 4.16	(D) 0.16	(E) none of these

»» Let X is a discrete uniform distribution with probability function

$f(x) = k, \qquad x = 1, 2, 3, 4$

(10) The value of k is

(A) 0.25	(B) 0.5	(C) 0.1	(D) 4	(E) none of these

»» Assume that the random variable X has the following probability density function (pdf):

$f(x) = 0.1$ for $0 < x < 10$.

(11) The mean of X (E(X)) is:

(A) 5	(B) 0.8	(C) 0.5	(D) 0.25	(E) none of these

(12) $P(X > 5)$ is:

(A) 1	(B) 0.8	(C) 0.6	(D) 0.5	(E) none of these

(13) $E(X^2)$ is:

(A) 0.33	(B) 33.3	(C) 333.3	(D) 5	(E) none of these

(14) V (X) is:

(A) 10	(B) 2.9	(C) 8.3	(D) 0.5	(E) none of these

(15) $P(X \geq 15)$ is:

(A) 1	(B) 0	(C) 0.25	(D) 0.15	(E) none of these

»» An audio amplifier contains five transistors. It has been ascertained that two of the transistors are faulty but it is not known which two. Two transistors are selected at random, and inspected. Then

(16) The probability that both the selected transistors are non-defective is:

(A) 0.40	(B) 0.50	(C) 0.36	(D) 0.30	(E) none of these

(17) The probability that both the selected transistors are defective is:

(A) 0.40	(B) 0.60	(C) 0.16	(D) 0.10	(E) none of these

(18) If X = the number of defective transistors found in the sample, then the mean of X is:

(A) 0.5	(B) 1.0	(C) 0.4	(D) 0. 8	(E) none of these

»»» Five terminals on an on-line computer system are attached to a communication line to the central computer system. The probability that any terminal is ready to transmit is 0.95. Then

(19) The probability that at least 4 will be ready is:

(A) 0.7738	(B) 0.2036	(C) 0.9774	(D0.0226	(E) none of these

(20) The mean of the number of ready terminals is:

(A) 4.75	(B) 0.2375	(C) 3.0	(D) 2.5	(E) none of these

(21) The variance of the number of ready terminals is:

(A) 4.75	(B) 0.2375	(C) 0.25	(D) 2.0	(E) none of these

»»» The number of traffic accidents that occurs on a particular road during a month follows a Poisson distribution with a mean of 8.

(22) The probability that less than two accidents will occur on that road during a randomly selected month is:

(A) 0.0331	(B) 0.0003	(C) 0.003	(D) 0.9997	(E) none of these

(23) The probability that exactly 4 accidents will occur on that road during a randomly selected 2 months is:

(A) 0.0331	(B) 0.003	(C) 0.0003	(D) 0.0573	(E) none of these

(24) The standard deviation for the number of accidents during a randomly selected month is:

(A) 8.00	(B) 4.00	(C) 2.83	(D) 2.00	(E) none of these

»»» The weights of the population of the Riyadh follows a normal distribution with mean 80 kg and standard deviation 5 kg, while the population of Damam follows a normal distribution with mean 75 kg and standard deviation 3 kg. If we select a random sample of 50 persons who live in Riyadh and another random sample of 36 persons who live in Damam then,

(25) The variance of the difference $\overline{X}_1 - \overline{X}_2$, $\sigma^2_{\overline{X}_1 - \overline{X}_2}$, is:

(A) 0.75	(B) 0.3770	(C) 0.7498	(D) 0. 2502	(E) none of these

(26) $P(4 < \overline{X}_1 - \overline{X}_2 < 6) =$

(A) 0.8749	(B) 0.3821	(C) 0.7498	(D) 0. 2523	(E) none of these

»» Assume that the average weight of a 6th grader is 80 pounds, with a standard deviation of 20 pounds. Suppose you draw a random sample of 25 students.

(27) The probability that the average weight of a sampled student is more than 75 pounds, assuming the population is normally distributed, is:

(A) 0.1056	(B) 0.8944	(C) 0.4013	(D) 0.5967	(E) none of these

(28) The average of 25 students is called a

(A) statistic	(B) parameter	(C) sample	(D) population	(E) none of these

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»» A sample of 25 observations is drawn from a normal population with mean = 40 and variance = 100.

(29) The mean of the sample mean, $\mu_{\bar{X}}$, is

(A) 10	(B) 100	(C) 6.32	(D) 40	(E) none of these

(30) The standard error of sample mean, $\sigma_{\bar{X}}$, is

(A) 10	(B) 100	(C) 40	(D) 2	(E) none of these

Good luck

(1) $X \sim Exp(\beta)$

$f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}} \quad , \quad x > 0$

$\beta = 5 \quad , \quad \mu_X = 5$

(2) $\sigma_X = \beta = 5$

(3) $P(\mu - 2\sigma < X < \mu + 2\sigma) = P(-5 < X < 5) = \int_{-5}^{5} f(x)\,dx$

$= \int_0^5 \frac{1}{5} e^{-\frac{x}{5}}\,dx = -e^{-\frac{x}{5}}\Big|_0^5$

$= 1 - e^{-1} = 0.632$

(4) $P(\mu - 2\sigma < X < \mu + 2\sigma)$

$= P(|X - \mu| < 2\sigma) \geq 1 - \frac{1}{2^2} = 0.75$

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(5) $X \sim N(\mu, \sigma) \qquad \mu = 128 \quad , \quad \sigma = 9$

$P(X \geq 110) = P(Z \geq \frac{110 - 128}{9}) = P(Z \geq -2)$

$= 1 - P(Z < -2) = 1 - 0.0228 = 0.9772$

(6) $P(X < 149) = P(Z < \frac{149 - 128}{9}) = P(Z < 2.33) = 0.9901$

(7) $P(X > a) = 0.5 \Rightarrow P(X \leq a) = 1 - 0.5$

$P(Z \leq \frac{a - 128}{\sigma}) = 0.5$

$\frac{a - 128}{\sigma} = 0 \qquad \underline{\underline{a = 128}}$

(8) $X \sim N(\mu, \sigma) \Rightarrow Z \sim N(0,1)$

$Z = \frac{X - \mu}{\sigma}$

$P(X < \mu + 2\sigma) = P(Z < \frac{\mu + 2\sigma - \mu}{\sigma}) = P(Z < 2)$

$= 0.9772$

(9) $P(X < 3) = 0.877$

$P(Z < \frac{3 - \mu}{1}) = 0.877 \qquad 3 - \mu = 1.16$

$\boxed{\mu = 1.84}$

(10) $f(x) = k \quad ; \quad x = 1, 2, 3, 4$

$$k = \frac{1}{4} = 0.25$$

(11) $f(x) = 0.1 \quad ; \quad 0 < x < 10$

$$\mu_X = E(X) = \frac{a+b}{2} = 5 \quad \left(\frac{0+10}{2}\right)$$

(12) $P(X > 5) = \int_5^{10} 0.1 \, dx = 0.1 [10 - 5] = 0.5$

(13) $\sigma_X^2 = Var(X) = \frac{(b-a)^2}{12} = \frac{(10-0)^2}{12} = 8.33$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$8.33 = E(X^2) - (5)^2 \qquad E(X^2) = 33.33$$

(14) $Var(X) = 8.33$

(15) $P(X \geq 15) = 0$

(16) $N = 5, \quad k = 2, \quad n = 2$

$$P(X=0) = \frac{\binom{2}{0}\binom{3}{2}}{\binom{5}{2}} = 0.3$$

(17) $P(X=2) = \frac{\binom{2}{2}\binom{3}{0}}{\binom{5}{2}} = 0.1$

(18) $\mu_X = n \cdot \frac{k}{N} = 2 * \frac{2}{5} = 0.8$

(19) $n = 5, \quad p = 0.95, \quad q = 0.05$

$$P(X \geq 4) = P(X=4) + P(X=5)$$

$$= \binom{5}{4}(0.95)^4(0.05) + \binom{5}{5}(0.95)^5$$

$$= 0.9774$$

(20) $\mu_X = np = 5 * 0.95 = 4.75$

(21) $\sigma_X^2 = npq = 5 * 0.95 * 0.05 = 0.2375$

(22) $X \sim Poisson(\mu)$, $\mu = \lambda t$

$\underline{\lambda = 8}$ $\mu = 8 * 1 = 8$

$P(X<2) = P(X=0) + P(X=1)$

$= e^{-8}\frac{8^0}{0!} + e^{-8}\frac{8}{1!} = 9e^{-8} = 0.003$

(23) $\underline{\lambda = 8}$ $\mu = 8 * 2 = 16$

$P(X=4) = e^{-16}\frac{16^4}{4!} = 0.0003$

(24) $\mu = 8$ $\sigma^2 = \lambda t = 8$ $\sigma = \sqrt{8} = 2.83$

(25) $X_1 \sim N(\mu_1, \sigma_1)$ $X_2 \sim N(\mu_2, \sigma_2)$

$\mu_1 = 80$, $\sigma_1 = 5$ $\mu_2 = 75$, $\sigma_2 = 3$

$n_1 = 50$ $n_2 = 36$

$\sigma^2_{\bar{X}_1 - \bar{X}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{5^2}{50} + \frac{3^2}{36} = 0.75$

(26) $P(4 < \bar{X}_1 - \bar{X}_2 < 6) = P(\bar{X}_1 - \bar{X}_2 < 6) - P(\bar{X}_1 - \bar{X}_2 < 4)$

$= P(Z < \frac{6-5}{0.866}) - P(Z < \frac{4-5}{0.866})$

$= P(Z < 1.15) - P(Z < -1.15)$

$= .8749 - .1251 = .7498$

(27) $X \sim N(\mu, \sigma)$ $\mu_X = 80$, $\sigma_X = 20$

$n = 25$, $\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ $\sigma_{\bar{X}} = \frac{20}{\sqrt{25}} = 4$, $\mu_{\bar{X}} = 80$

$P(\bar{X} > 75) = 1 - P(\bar{X} \leq 75)$

$= 1 - P(Z \leq \frac{75-80}{4}) = 1 - P(Z \leq -1.25)$

$= 1 - 0.1056 = 0.8944$

(28) $\bar{X}$ is a statistic

(29) $\mu_{\bar{X}} = \mu_X = 40$

(30) $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{10}{\sqrt{25}} = \frac{10}{5} = 2$