Department of Statistics and Operations Research
College of Science
King Saud University

Second-Semester 1428/1429 Probability and Statistics for Engineering
Second Mid-term Exam Time: 90 minutes Total 15 Marks

Student name: Student number:

Instructor: Dr. Section Number:

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<tr><td>4</td><td></td><td>13</td><td></td><td>22</td><td></td><td colspan="2" rowspan="6">ملاحظه:
تنقل رموز الإجابات الصحيحه
A or B or C or D
بخط واضح
وحروف كبيره
Capital NOT small
و ممنوع الكشط
يغلق الجوال تماماً</td></tr>
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Q1.

A television of manufacturer A have a mean life time of 12 years with a standard deviation 0f 1.5 years while that of manufacturer B have a mean life time of 11 years with standard deviation of 0.9 years. Assuming that life time for each of these manufacturers A and B is normally distributed random variable. If we select a random sample of size 100 unites from each of the manufactures. Then:

(1) $P(\overline{X}_A - \overline{X}_B > 1.5)$ will be:

(A) 0.1170
(B) 0.0021
(C) 0.8930
(D) otherwise

(2) while $P(-0.5 \leq \overline{X}_A - \overline{X}_B < 0.5)$ is

(A) 0.4916
(B) 0.1265
(C) 0.8930
(D) Otherwise

Q2.

(3) The mean and standard deviation of the standard normal curve are

(A) $\mu = 0$ *and* $\sigma = 0$
(B) $\mu = 1$ *and* $\sigma = 1$
(C) $\mu = 0$ *and* $\sigma = 1$
(D) none of the above

(4)which of the following are characteristics of the normal curve:

(A) the area under the curve is 1
(B) the graph is symmetric about the mean
(C) the standard deviation determines the width of the curve
(D) all of the above

Q3. a random committee of size 3 is selected from 4 doctors 2 nurses. Let a random variable X be the number of doctors on the committee. Then

(5) $P(2 \leq X \leq 3)$ will be:

(A) 0.75
(B) 0.25
(C) 0.80
(D) 0.50

Q4.

If it is known that the probability distribution of a random variable X is normal with mean μ and variance σ^2, then the exact value of

(6) $p(\mu\text{-}3\sigma < X < \mu\text{+}3\sigma)$ is

(A) 0.8889
(B) 0.0026
(C) 0.1111
(D) 0.9974

Q5.

Given the normally distributed variable X with mean 18 and variance 6.25, then

(7) $P(X < 15)$ will be

(A) 0.8889
(B) 0.0026
(C) 0.1151
(D) 0.9974

(8) the value of K such that $P(X > K) = 0.7764$ will be

(A) 16.1
(B) 0.0026
(C) 0.1111
(D) 0.9974

Q6.

If a set of observations are normally distributed, then

(9) the percent of these observations differ from the mean by less than 1.3σ will be
(A) 80.64 %
(B) 19.36 %
(C) 56.99 %
(D) 99.11 %

Q7. Suppose that we have a population with mean 5.3 and variance 0.81, then
(10) the probability that the average of random sample of 36 will exceed 5.5 is
(A) 0. 9082
(B) 0.0918
(C) 0.1151
(D) otherwise

Q8.On the average a certain inspection results is 3 traffic accidents per month in an intersection. Then,

(11) the probability that in any given month at this intersection exactly 5 accidents will be occurred is
(A) 0.01
(B) 0.8
(C) 0.1
(D) 0.008

(12) the probability that at least 2 accidents will be occurred is
(A) 0.01
(B) 0.8
(C) 0.1
(D) 0.008

Q9
Assume that the random variable X has the following uniform distribution:

$$f(x)=\begin{cases}3 & ,\frac{2}{3}<x<1\\ 0 & ,otherwise\end{cases}\quad ;\text{ then}$$

(13) $P(X > 1.25)$ will be
(A) 0
(B) 1
(C) 0.5
(D) 0.33

(14) the variance of X is
(A) 0.6944
(B) 0.0093
(C) 0.5000
(D) 0.3333

Q10.
A multiple choice exam consists of 10 questions. Each question has 4 choices one of them is correct. A person answers this exam by pure guessing. Let X = number of correct answers this persons gets. If a person is selected at random from this group, then

(15) the $P(X \geq 1)$ is
(A) 0.0115
(B) 0.9437
(C) 0.0577
(D) otherwise

(16) The standard deviation of X is,
(A) 4
(B) 2
(C) 1.37
(D) otherwise

Q11.
Suppose that X is the number of patients admitted to the intensive care unite at the university hospital per day. If the average number of persons

admitted to the intensive care unite per day is three. Then the probability that in a given day there will be

(17) exactly two admissions in a given day is
(A) 0.224
(B) 0.6667
(C) 0.8333
(D) 0.1389

(18) at most two admissions in a given day, will be
(A) 0.6667
(B) 0.423
(C) 0.8333
(D) 0.1389

(19) at most two admissions in a period of two days is

(A) 0.6667
(B) 0.062
(C) 0.8333
(D) Otherwise

Q12.

Suppose that the length of a telephone call (in minutes) is exponentially distributed with an average (mean) value of 5. Then,

(20) the probability that the call lasts between two and seven minutes is

(A) 0.6703
(B) 0.2466
(C) 0.4237
(D) otherwise

(21) the standard deviation of the length of any telephone call will be
(A) 5
(B) 2
(C) 25
(D) 1.79

Q13.

The heights (in inches) of women, in a certain population, are normally distributed with a mean of 63.6 and a standard deviation of 2.5; then

(22) the percentage of women with heights greater than 66 inches will be

(A) 0.8315%
(B) 16.85%
(C) 0.1685%
(D) otherwise

Q14.
Given $Z \sim N(0,1)$, then
(23) the area under curve to the right of Z= -8/3 is:
(A) 0.9962
(B) 0.9477
(C) 0.0038
(D) 0.5219

(24) the value of k such that $P(Z \geq k)=0.3015$ will be:
(A) -0.52
(B) 1.11
(C) 2.11
(D) 0.52

(25) while k which satisfies $P(0.93 < Z < k) = 0.0427$ will be
(A) -0.52
(B) 1.11
(C) 2.11
(D) otherwise

Q15.

Suppose that 20% of the students in a certain university smoke cigarettes. A random sample of size 10 students is selected from this university. Let $\hat{p}$ be sample proportion of smokers in the sample, then

(26) $V(\hat{p}) = \sigma^2_{\hat{p}} =$

(A) 0.1170
(B) 0.1265
(C) 0.8930
(D) 0.0160

(27) $P(\hat{p} > 0.35) =$

(A) 0.1170
(B) 0.1265
(C) 0.8930
(D) 0.0160

Q16. Given the random variable T has the t-distribution with υ degrees of freedom then

(28) the t-value with $\upsilon = 16$ degrees of freedom that leaves an area of 0.025 to the left will be:

(A) 2.120
(B) 2.382
(C) -2.120
(D) -2.382

(29) Find the k value which satisfies $P(-2.069 < T < k) = 0.965$, $with\ \upsilon = 23$

A 0.10
B 0.05
C 0.01
D 0.025

(30) with $\upsilon = 12$, the value of $P(-1.356 < T < 2.179)$ will be:

(A) 0.875
(B) 0.125
(C) 0.025
(D) otherwise

Good luck

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Population A

$X_A \sim N(12, 1.5)$

$n_A = 100$

Population B

$X_B \sim N(11, 0.9)$

$n_B = 100$

$$\bar{X}_A - \bar{X}_B \sim N\left(\mu_A - \mu_B, \sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}\right)$$

$$\sim N(1, 0.175)$$

(1) $P(\bar{X}_A - \bar{X}_B > 1.5) = P\left(Z > \frac{1.5 - 1}{0.175}\right)$

$P(Z > 2.86) = 1 - P(Z < 2.86)$

$= 1 - 0.9979 = 0.0021$ B

(2) $P(-0.5 \leq \bar{X}_A - \bar{X}_B < 0.5)$

$= P(-8.57 \leq Z < -2.86)$

$= P(Z < -2.86) - P(Z \leq -8.57)$

$= 0.0021 - 0 = 0.0021$ D

Q2

(3) Std. Normal curve $\mu = 0, \sigma = 1$ C

(4) D

Q3 $X \sim hyp(n, N, K)$

$$f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \quad x = 0, 1, 2, \ldots, n$$

4 D	2 NS

$N = 6$

3

(5) $P(2 \leq X \leq 3) = P(X = 2) + P(X = 3)$

$$= \frac{\binom{4}{2}\binom{2}{1} + \binom{4}{3}\binom{2}{0}}{\binom{6}{3}} = 0.80$$ C

Q4 $X \sim N(\mu, \sigma^2)$

(6) $P(\mu - 3\sigma < X < \mu + 3\sigma)$

$= P(-3 < Z < 3) = P(Z < 3) - P(Z < -3)$ D

$= 0.9987 - 0.0013 = \underline{0.9974}$

Q5 $X \sim N(\mu, \sigma)$, $\mu = 18$, $\sigma = \sqrt{6.25} = 2.5$

(7) $P(X < 15) = P(Z < \frac{15 - 18}{2.5}) = P(Z < -1.2) = 0.1151$ C

(8) $P(X > K) = 0.7764$

$P(X \leq K) = 0.2236$

$P(Z \leq \frac{K - 18}{2.5}) = 0.2236$ $\qquad \frac{K-18}{2.5} = -0.76$

$\underline{K = 16.1}$ A

Q6

(9) $P(\mu - 1.3\sigma < X < \mu + 1.3\sigma)$

$= P(-1.3 < Z < 1.3) = P(Z < 1.3) - P(Z < -1.3)$

$= 0.9032 - 0.0968$

$= 0.8064 = \underline{80.64\%}$ A

Q7 $\underline{\mu = 5.3}$, $\sigma^2 = 0.81$ $\underline{\sigma = 0.9}$

(10) $n = 36$ Large n assume Normal distribution

$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ $\qquad \bar{X} \sim N(5.3, 0.15)$

$P(\bar{X} > 5.5) = P(\bar{Z} > \frac{5.5 - 5.3}{0.15}) = 1 - P(\bar{Z} < 1.33)$

$= 1 - 0.9082$

$= 0.0918$ B

Q8 $X \sim \text{Poisson}(\mu)$, $\mu = 3$

(11) $P(X=5) = e^{-3} \cdot \frac{3^5}{5!} = 0.1$ C

(12) $P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$

$= 1 - e^{-3}\left[\frac{3^0}{0!} + \frac{3}{1!}\right] = 0.8$ B

Q9

$$f(x) = \begin{cases} 3 & \frac{2}{3} < x < 1 \\ 0 & \text{o.w.} \end{cases} \qquad X \sim U\left(\tfrac{2}{3}, 1\right)$$

(13) $P(X > 1.25) = 0$ A

(14) $\text{Var}(X) = \frac{(b-a)^2}{12} = 0.0093$ B

Q10 $X \sim \text{binomial}(n, p)$

$n = 10$, $p = 0.25$

(15) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X=0)$

$= 1 - \binom{10}{0}(0.25)^0(0.75)^{10}$

$= 0.9437$ B

(16) $\text{Var}(X) = npq = 1.875$

std. dev. $\sigma = 1.37$ C

Q11 $X \sim \text{Poisson}(\mu)$ $\mu = 3$

(17) $P(X=2) = e^{-3} \cdot \frac{3^2}{2!} = 0.224$ A

(18) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= e^{-3}\left[\frac{3^0}{0!} + \frac{3}{1!} + \frac{3^2}{2!}\right]$

(19) $\mu = 6$

$P(X \leq 2) = e^{-6}\left[\frac{6^0}{0!} + \frac{6}{1!} + \frac{6^2}{2!}\right]$

Q12 $X \sim Exp(\beta)$ $\quad \beta = 5$

$$f(x) = \frac{1}{5} e^{-\frac{x}{5}} \quad , x > 0$$

(20) $P(2 < X < 7) = \int_2^7 \frac{1}{5} e^{-\frac{x}{5}} dx = -e^{-\frac{x}{5}} \Big|_2^7$

$$= e^{-0.4} - e^{-1.4}$$

(21) $Var(X) = \beta^2 \Rightarrow \sigma = \beta = 5$ (A)

Q13 $X \sim N(\mu, \sigma)$ $\quad \mu = 63.6 \; , \; \sigma = 2.5$

(22) $P(X > 66) = P(Z > \frac{66 - 63.6}{2.5})$

$$= 1 - P(Z \leq 0.96)$$

$$= 1 - 0.8315 = 0.1685$$

$$= 16.85\ \%$$ (A)

Q14 $Z \sim N(0,1)$

(23) $P(Z > -\frac{8}{3}) = 1 - P(Z \leq -2.67)$

$$= 1 - 0.0038 = 0.9962$$ (A)

(24) $P(Z \geq K) = 0.3015$

$P(Z < K) = 0.6985$ $\quad K = 0.52$ (D)

(25) $P(0.93 < Z < K) = 0.0427$

$P(Z < K) - P(Z < 0.93) = 0.0427$

$P(Z < K) = 0.0427 + 0.8238 = 0.8665$

$K = 1.11$ (B)

Q15 $p = 0.2$, $n = 10$

$$\hat{P} \sim N(P, \sqrt{\frac{pq}{n}})$$

$$E(\hat{P}) = P = 0.2$$

(26) $Var(\hat{P}) = \frac{pq}{n} = 0.016$ (D)

(27) $\hat{P} \sim N(0.2, 0.13)$

$$P(\hat{P} > 0.35) = 1 - P(\hat{P} \leq 0.35)$$
$$= 1 - P(\hat{Z} \leq 1.19)$$
$$= 1 - 0.883 = 0.117$$ (A)

Q16 $T \sim t(\nu)$

(28) $\nu = 16$ $\boxed{t_{1-\alpha} = -t_\alpha}$

$P(T < t_\alpha) = 0.025$ $P(T > t_\alpha) = 0.875$

$t_{1-\alpha} = t_{0.025} = 2.12$ $t_\alpha = -2.12$ (C)

(29) $P(-2.069 < T < k) = 0.965$, $\nu = 23$

$$P(T < k) - P(T < -2.069) = 0.965$$

$$P(T < k) = 0.965 + P(T > 2.069)$$
$$= 0.965 + 0.025 = 0.99$$

$$P(T > k) = 0.01$$

$$k = 2.5$$

(30) $P(-1.356 < T < 2.179)$ $\nu = 12$

$$P(T < 2.179) - P(T < -1.356)$$
$$= 1 - P(T > 2.179) - P(T > 1.356)$$
$$= 1 - 0.025 - 0.1 = 0.875$$ (A)