Hiit 1 ; actu 464.

**Question 1.** (2+2=4 marks)
Suppose that an insured has an exponential utility function with parameter $\alpha$. Suppose the risk $X$ has a Poisson distribution with parameter $\theta$.

a) Compute the maximum premium $P^+$ to be accepted for the risk $X$?
b) Compute $P^+$ using the approximation formula?

a) $P^+ = \frac{1}{\alpha} \ln(m_X(\alpha))$ ; $m_X(t) = e^{\theta(e^t - 1)}$

$= \frac{1}{\alpha} \theta (e^{\alpha} - 1)$

b) $P^+ = \mu + \frac{1}{2}\sigma^2 r(w-\mu)$ ; $r(x) = \frac{\alpha}{2}$, $\mu = \sigma^2 = \theta$.

$= \theta + \frac{1}{2}\theta\alpha$

$= \theta\left(1 + \frac{\alpha}{2}\right).$

**Question 2.** (2 marks)
Compute the stop-loss premium for $d = 4$ and a risk $X \sim Uniform(2,6)$?

$E(X-4)_+ = \int_4^6 \frac{1}{4}(x-4)\,dx$

$= \frac{1}{4}\,\frac{1}{2}(x-4)^2 \Big|_4^6 = \frac{1}{2}.$



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**Question 3.** (2+2=4 marks)

Let consider the random variable $X$ having the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & if \quad x < 0 \\ c(x^2+1) & if \quad 0 \le x < 3 \\ 1 & if \quad x \ge 3 \end{cases}$$

with $c = 1/15$.

a) Compute the mean of $X$.

b) Compute the mgf of $X$.

a) $E(x) = \int x\, dF(x) = \int_{-\infty}^{0^-} + \int_{0^-}^{0} + \int_{0}^{3^-} + \int_{3^-}^{3} + \int_{3}^{\infty}$

$= 0 + 0 + \int_0^3 x \cdot 2cx\, dx + (1 - \frac{10}{15}) * 3 + 0$

$= \frac{2cx^3}{3}\Big|_0^3 + 1 = \cancel{2c}\; 18c + 1 = 33/15.$

b) $m_X(t) = \int e^{tx}\, dF(x) = \int_{-\infty}^{0^-} + \int_{0^-}^{0} + \int_{0}^{3^-} + \int_{3^-}^{3} + \int_{3}^{\infty}$

$= 0 + 1 * c + 2c \int_0^3 x e^{tx}\, dx + e^{3t}(1 - \frac{10}{15}) + 0.$

$= c + 2c \left( \frac{x}{t} e^{tx}\Big|_0^3 - \frac{1}{t^2} e^{tx}\Big|_0^3 \right) + \frac{1}{3} e^{3t}$

$= c + 2c \left( \frac{3e^{3t}}{t} - \frac{1}{t^2} e^{3t} + \frac{1}{t^2} \right) + 1/3\, e^{3t}.$

**Question 4.** (3+1=4 marks)

Suppose $X$ and $Y$ are two independent discrete random variables with $f_X(x) = x/4$ for $x = 1,3$ and $f_Y(y) = y/6$ for $y = 1,2,3$.

a) Find the mass function of $X + Y$.

b) Compute $P(X + Y \ge 3)$.

a)

| $x \backslash y$ | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 3 | 4 | 5 | 6 |

| $x$ | $f_X$ | $f_Y$ | $f_S$ |
|---|---|---|---|
| 1 | 1/4 | 1/6 | 1/24 |
| 2 | | 1/3 | 1/12 |
| 3 | 3/4 | 1/2 | $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$ |
| 4 | | | 1/4 |
| 5 | | | 3/8 |
| 6 | - | - | - |

b) $P(X+Y \ge 3) = 1 - P(X+Y \le 2)$

$= 1 - 1/24 = 23/24$



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**Question 5.** (2+2+2=6 marks)

a) Suppose $X$ and $Y$ are two independent random variables with $X \sim Negative\_Binomial(r, 0.5)$ and $Y \sim Negative\_Binomial(s, 0.5)$. Prove that $X + Y \sim Negative\_Binomial(r + s, 0.5)$.

b) Suppose $X_1, X_2, \dots X_n$ are iid with common distribution $X \sim Negative\_Binomial(r, 0.5)$. Find the distribution of $S = X_1 + \cdots + X_n$.

c) Suppose that $n = 1{,}000$ people take out a one year insurance policy with risk payment X distributed as a Negative Binomial with parameters $r = 4$ and $p = 0.5$. Calculate the probability that the total payment is at least 4,200, using the normal approximation.

a) $m_X(t) = \left(\frac{0.5}{1-0.5e^t}\right)^r$, $m_Y(t) = \left(\frac{0.5}{1-0.5e^t}\right)^s$

$m_{X+Y}(t) = m_X(t) \cdot m_Y(t) = \left(\frac{0.5}{1-0.5e^t}\right)^{r+s}$.

b) $S \sim Neg\text{-}Bin(nr; 0.5)$.

c) $X \sim Neg\text{-}Bin(4; 0.5) \rightarrow S \sim Neg\text{-}Bin(4{,}000; 0.5)$

$\mu = 4{,}000$
$\sigma^2 = 8{,}000$

$P(S \geq 4{,}2000) = 1 - P\left(\frac{S-\mu}{\sigma} \leq \frac{4200-4000}{\sqrt{8000}}\right)$

$\simeq 1 - \phi(2.23) = 1 - 0.9871 = 0.0129$

**Question 6.** (5 marks)

Suppose an insurer is looking for an optimal reinsurance for a portfolio consisting of 8,000 one-year life insurance policies that are grouped as follows:

| Insured amount | Number of policies |
|---|---|
| 1 | 4,000 |
| 3 | 4,000 |

The probability of dying within one year is 0.03 for each insured, and the policies are independent. Calculate the minimum capital B that covers losses $S$ with probability 95%, for a priority $d = 1$ and using the Normal Power (NP) approximation.

Insurance:

| Amount | Nber |
|---|---|
| 1 | 8,000 |

Reinsurance

| amount | Nber |
|---|---|
| 2 | 4,000 |

$S_1 = X_1 + \dots + X_{8000}$
$X \sim Ber(0.03)$

$S_2 = Y_1 + \dots + Y_{4{,}000}$
$Y \sim 2\,Ber(0.03)$

$S = S_1 + E(S_2)$.

$\boxed{\mu_2 = 4{,}000 * 2 * 0.03 = 240}$

$\mu_1 = 8000 * 0.03 = 240$

$\boxed{\mu = \mu_1 + \mu_2 = 480}$

$\sigma^2 = \sigma_1^2 = 8000 * 0.03 * 0.97 = 232.8$

$K_1(3) = K_1(S_1) = 8000 * 0.03 * 0.97 * 0.94 = \cancel{0.672}\ 218.83$



$\gamma = \frac{K_1(S)}{\sigma^3} = 0.06$

$B = \mu + \sigma\left(s + \frac{\gamma}{6}(s^2 - 1)\right)$, $s = \phi^{-1}(0.95) = 1.645$

$= \cancel{486.25}\ 505,36$

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