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STAT 324
First Midterm Exam
Second Semester
1430 – 1431 H

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- Mobile Telephones are <u>not allowed</u> in the classrooms.
- Time allowed is <u>90 minutes</u>.
- Answer all questions.
- Choose the nearest number to your answer.
- WARNING: Do not copy answers from your neighbors. <u>They have different questions forms.</u>
- For each question, put the code of the correct answer in the following table beneath the question number:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	C	C	D	B	A	D	B	A	D

11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	B	D	C	A	C	A	B	B	C

21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	A	C	B	D	C	A	D	D	A

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Suppose that the error in the reaction temperature, in ^{0}C , for a controlled laboratory experiment is a continuous random variable X having the function

$$f(x)=\begin{cases}\frac{8}{x^3}, & x>2\\ 0, & \text{elsewhere},\end{cases}$$

then:

(1)	$P(X<4)$							
	(A)	**0.75**	(B)	3.0	(C)	0.50	(D)	0.15
(2)	$P(-1<X<4)$							
	(A)	3.0	(B)	0.15	(C)	**0.75**	(D)	0.5
(3)	$P(X\geq 5)$							
	(A)	1.0	(B)	0.15	(C)	**0.16**	(D)	0.5
(4)	The expected value of X; E(X) equals							
	(A)	2.0	(B)	1.0	(C)	8.0	(D)	**4.0**

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An investment firm offers its customers municipal bands that mature after different numbers of years. Given that cumulative distribution function of X, the number of years to maturity for a randomly selected bond is:

$$F(x)=\begin{cases}0 & x<1\\ 0.24 & 1\leq x<3\\ 0.56, & 3\leq x<5\\ 1, & x\geq 5\end{cases}$$

(5)	$P(X=5)$ equals to							
	(A)	0.76	(B)	**0.44**	(C)	0.56	(D)	0.20
(6)	$P(X>2)$							
	(A)	**0.76**	(B)	0.56	(C)	0.50	(D)	0.20
(7)	$P(1.5<X<5)$							
	(A)	0.2	(B)	0.76	(C)	0.56	(D)	**0.32**

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Suppose that $P(A_1)=0.4$, $P(A_1\cap A_2)=0.2$, $P(A_3|A_1\cap A_2)=0.75$, then:

(8)	$P(A_2|A_1)$ equals to							
	(A)	0.00	(B)	**0.50**	(C)	0.1	(D)	0.2
(9)	$P(A_1\cap A_2\cap A_3)$ equals to							
	(A)	**0.15**	(B)	0.75	(C)	1.0	(D)	0.2

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A certain group of adults are classified according to sex and their level of education as given by the following table:

Sex Education	Female	Male
College	17	22
Secondary	45	38
Elementary	50	28

If a person is selected at random from this group, then

(10)	The probability that the person is female is:							
	(A)	0.44	(B)	0.50	(C)	0.28	(D)	**0.56**
(11)	The probability that the person is female and has an elementary education is:							
	(A)	0.64	(B)	**0.25**	(C)	0.45	(D)	0.50

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Suppose that a certain institute offers two training programs T_1 and T_2. In the last year, 100 and 200 trainees were enrolled for programs T_1 and T_2, respectively. From the past experience it is known that the passing probabilities are 0.75 for the program T_1 and 0.80 for the program T_2. Assume that at the end of the last year we selected a trainee at random from this institute.

(12)	The probability that the selected trainee passed the program equals to							
	(A)	0.53	(B)	**0.78**	(C)	0.50	(D)	0.25
(13)	What is the probability that the selected trainee has been enrolled in the program T_2 given that he passed the program							
	(A)	0.80	(B)	0.32	(C)	0.78	(D)	**0.68**

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If $P(A)=0.9$, $P(B)=0.6$, and $P(A^C \cap B)=0.1$, then:

(14)	$P(A \cap B)$ equals to							
	(A)	0.30	(B)	0.40	(C)	**0.50**	(D)	0.20
(15)	$P(A \cup B)^C$ equals to							
	(A)	**0.00**	(B)	1.00	(C)	0.50	(D)	0.15
(16)	$P(A^c \mid B)$ equals to							
	(A)	0.10	(B)	0.50	(C)	**0. 17**	(D)	0.011
(17)	$P(B \mid A^c)$ equals to							
	(A)	**1.00**	(B)	0.011	(C)	0.50	(D)	0.017

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If $P(A)=0.8$, $P(B)=0.5$, and $P(A\ U B)=0.9$, then:

(18)	The two events A and B are							
	(A)	dependent	(B)	**independent**	(C)	disjoint	(D)	Mutually exclusive

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If the function $f(x) = C(x^2 + 3)$ for x = 0, 1, 2 can serve as a probability distribution of the discrete random variable X.

(19)	The value of C equals to							
	(A)	14	(B)	**0.071**	(C)	12	(D)	0.032

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Suppose that we have probability function $f(x) = 0.1x$, for x = 1, 2, 3, 4. Then

(20)	P(X >2) equals to							
	(A)	0.3	(B)	0.1	(C)	**0.7**	(D)	0.9
(21)	The expected value of X equals							
	(A)	**3.0**	(B)	2.5	(C)	0.25	(D)	0.5
(22)	The Variance of X equals							
	(A)	**1.0**	(B)	3.54	(C)	1.25	(D)	0.5

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If the random variable X has probability density

$$f(x) = \begin{cases} \dfrac{x^2}{3}, & k < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

(23)	Then the value of k equals							
	(A)	0.44	(B)	0.40	(C)	**-1.0**	(D)	0.23

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If the random variable X has probability density

$$f(x) = \begin{Bmatrix} 1 + x, & -1 < x < 0 \\ 1 - x & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{Bmatrix}$$

(24)	P(X < 0.5) equals to							
	(A)	0.5	(B)	**0.875**	(C)	0.375	(D)	0.75
(25)	P(X = 0.2) equals to							
	(A)	1.2	(B)	0.5	(C)	0.8	(D)	**0**

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The cumulative distribution function F(x) of a continuous random variable X is as follows:

$$F(x)=\begin{cases}0, & x \le -1\\ \frac{x^3+1}{9}, & -1 < x < 2\\ 1 & x \ge 2\end{cases}$$

(26)	$P(-0.5 < X < 1.5)$ equals to							
	(A)	0.30	(B)	0.40	(C)	**0.39**	(D)	0.20
(27)	$P(X \ge 0.6)$ equals to							
	(A)	**0. 86**	(B)	0.14	(C)	0.50	(D)	0.15

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A random variable 'X' has E(X) = 2 and $E(X^2) = 8$. Another random variable 'Y' is related with X as follows:

Y = (3X + 5) /2.

(28)	The mean of Y is:							
	(A)	2.0	(B)	6.0	(C)	8.5	(D)	**5.5**
(29)	The Variance of Y is:							
	(A)	4.0	(B)	8.5	(C)	6.0	(D)	**9.0**

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A random variable 'X' has E(X) = 2, and variance = 4.

(30)	Then by Chebychev theorem, P(-1 < X < 5) is							
	(A)	**≥ 5/9**	(B)	≥ 4/9	(C)	≤ 5/9	(D)	≤ 4/9

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(1) $P(X<4) = \int_{-\infty}^{4} f(x)\, dx = \int_{2}^{4} \frac{8}{x^3}\, dx$

(2) $P(-1<X<4) = \int_{-1}^{4} f(x)\, dx = \int_{2}^{4} \frac{8}{x^3}\, dx$

(3) $P(X \geq 5) = 1 - P(X<5) = 1 - \int_{2}^{5} \frac{8}{x^3}\, dx$

(4) $E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$

$= \int_{2}^{\infty} \frac{8}{x^2}\, dx = -\frac{8}{x}\Big|_{2}^{\infty} = 0-(-4)$

(5) $P(X=5) = F(5) - F(3) = 1 - 0.56 = 0.44$

(6) $P(X>2) = 1 - P(X \leq 2)$

$= 1 - F(2) = 1 - 0.24$

or $= P(X=3) + P(X=5)$

$F(3) - F(1) + F(5) - F(3)$

(7) $P(1.5 < X < 5) = F(5) - F(1.5) - f(5)$

$= 1 - 0.24 - 0.44$

or $= P(X=3)$

(8) $P(A_2 | A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)}$

(9) $P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P(A_2 | A_1)\, P(A_3 | A_1 \cap A_2)$

(10) $P(F) = P(F\cap C) + P(F\cap S) + P(F\cap E)$

$= \dfrac{17 + 45 + 50}{200} = \dfrac{112}{200} = 0.56$

(11) $P(F\cap E) = \dfrac{50}{200} = 0.25$

(12) $P(S) = P(T_1)\,P(S|T_1) + P(T_2)\,P(S|T_2)$

(13) $P(T_2|S) = \dfrac{P(T_2)\,P(S|T_2)}{P(S)}$

(14) $P(A\cap B) = P(B) - P(A^c\cap B)$

(15) $P(A\cup B)^c = 1 - P(A\cap B)$

(16) $P(A^c|B) = \dfrac{P(A^c\cap B)}{P(B)}$

(17) $P(B|A^c) = \dfrac{P(A^c\cap B)}{P(A^c)}$

(18) $P(A) = 0.8$, $P(B) = 0.5$, $P(A\cup B) = 0.9$

$\Rightarrow P(A\cap B) = 0.4 = P(A)\,P(B)$

A, B are independent

(19) $\sum_x f(x) = 1$ $\quad c(0+3) + c(1^2+3) + c(2^2+3) = 1$

$14c = 1$ $\quad c = \frac{1}{14}$

(20) $f(x) = 0.1\, x \quad , \quad x = 1, 2, 3, 4$

$$P(X > 2) = P(X=3) + P(X=4)$$
$$= f(3) + f(4) = 0.3 + 0.4 = 0.7$$

(21) $\mu = \sum x\, f(x)$

$$= (1)(0.1) + (2)(0.2) + (3)(0.3) + 4(0.4)$$

(22) $\sigma^2 = \Sigma(X^2) - \mu^2$

$$\Sigma(X^2) = (1^2)(0.1) + (2^2)(0.2) + (3^2)(0.3) + (4^2)(0.4)$$

(23) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\int_{K}^{2} \frac{x^2}{3}\, dx = 1 \Rightarrow \left.\frac{x^3}{9}\right|_K^2 = 1$$

$$\frac{8}{9} - \frac{K^3}{9} = 1 \Rightarrow \underline{K = -1}$$

(24) $P(X < 0.5) = \int_{-\infty}^{0.5} f(x)\, dx$

$$= \int_{-1}^{0} (1+x)\, dx + \int_{0}^{0.5} (1-x)\, dx$$

$$= \left. x + \frac{x^2}{2} \right|_{-1}^{0} + \left. x - \frac{x^2}{2} \right|_{0}^{0.5}$$

$$= 0 + 1 - 0.5 + 0.5 - 0.125 - 0$$

(25) $P(X = 0.2) = 0$

(26) $P(-0.5 < X < 1.5)$

$$= F(1.5) - F(-0.5)$$

$$= \frac{(1.5)^3+1}{9} - \frac{(-0.5)^3+1}{9} = \frac{3.5}{9} = 0.39$$

(27) $P(X \geq 0.6) = 1 - P(X < 0.6)$

$$= 1 - F(0.6)$$

$$= 1 - \frac{(0.6)^3+1}{9} = 0.86$$

(28) $E(Y) = E(\frac{3}{2}X + \frac{5}{2})$

$$= \frac{3}{2}E(X) + \frac{5}{2}$$

(29) $Var(Y) = \frac{9}{4} Var(X)$

(30) $P(-1 < X < 5)$

$P(-3 < X-2 < 3) \geq 1 - \frac{1}{K^2} = 1 - \frac{4}{9} = \frac{2}{9}$

$(-\frac{3}{2})(2) < X-2 < (\frac{3}{2})(2)$ $K = \frac{3}{2}$