

Student's Name	Student's ID	Group No.

Question No.	I	II	III	IV	V	Total
Mark						

[I] Determine whether the following is **True** or **False**. **Justify** your answer. [4 Points]

1. If p^* approximates $p = \pi^5$ with relative error at most 10^{-3} , then $p^* \in [\pi^5 - 10^{-3}, \pi^5 + 10^{-3}]$ ()

2. The rate of convergence for $\frac{n^2+4}{n^3+n}$ as $n \rightarrow \infty$ is $O(\frac{1}{n^2})$ ()

3. If $f(h) = \frac{\sin h}{h}$, then $f(h) = 1 + O(h^2)$ ()

4. If 3-digit rounding is used to evaluate $g(x) = x^3 + 4.2x$ at $x = 1.34$, then $g(1.34) = 8.02$ ()

OVER

[II] Use the Bisection method to **find the root** of $\sin(x) - e^{-x}$ on $[0, 1]$ accurate to within 10^{-2} [3 Points]

[III] **Approximate** $\sqrt{7}$ using Newton's method with $p_0 = 2.5$ and accuracy 10^{-4} (Hint: Let $f(x) = x^2 - 7$) [3 Points]

OVER

[IV] Let $f(x) = x^3 - x - 1$ [5 Points]

- i. If $f(p) = 0$, **show** that $g_1(x) = \sqrt[3]{x+1}$ and $g_2(x) = x^3 - 1$ both have a fixed point at p
- ii. **Prove** that g_1 has a unique fixed point on $[1, 2]$
- iii. Starting with $p_0 = 1.4$, **use** a fixed-point iteration on g_1 to determine the root p of f on $[1, 2]$ accurate to within 10^{-3}

OVER

[V] Let $f(x) = (x - 2)^2 - \ln x$ for $1 \leq x \leq 2$. Starting with $p_0 = 1$ and $p_1 = 1.5$, **find** p_3 by [5 Points]

- i. The Secant method
- ii. The method of False Position
- iii. Which method gives a better approximation for the root of f ? **Justify** your answer