Questions	Q(I) (1—8)	Q(I) (9-10)	Q(II) (A)	Q(II) (B)	Q(III) (A)	Q(III) (B)	Q(IV) (A)	Q(IV) (B)	Total
Marks	8	2	5.5	5.25 6	6.5	3.5	4.5	5	46.25

≈23.25

Question I:

Question Number	1	2	3	4	5	6	7	8	9	10
Answer	b	a	c	b	d	c	a	b	c	d

Choose the correct answer, then fill in the table above: (5 points)

1- For the equivalence relation on $\mathbb{Z}$ defined by $a \equiv b \ (mod\ 8)$:

(a) [2]=[8].
(b) [2]=[10].
(c) [1]=[8].
(d) [2]=[-10].

?

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refl + symm+trans

2- If {{0}, {-1}, {1}, {2}} is a partitions of {-1, 0, 1, 2}, then the ordered pairs in the equivalence relations produced by this partitions is:

(a) {(0,0), (1,1), (-1,-1), (2,2)}.
(b) {(-1,1), (1,1), (0,1), (0,0), (1,-1), (1,2), (2,1), (2,2)}.
(c) {(-1,-1), (-1,0), (-1,1), (-1,2), (0,-1), (0,0), (0,1), (0,2), (1,-1), (1,0), (1,1), (1,2), (2,-1), (2,0), (2,1), (2,2)}.
(d) None of the previous.

a or c

3- The directed graph below represented a relation R which is:

$R = \{(a,a), (a,c), (b,b), (b,a), (c,c), (c,b)\}$

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 1 \\ b & 1 & 1 & 0 \\ c & 0 & 1 & 1 \end{array}$$

(a) Reflexive, symmetric, antisymmetric and not transitive.
(b) Reflexive and transitive but not symmetric and not antisymmetric.
(c) Reflexive and antisymmetric only.
(d) Reflexive and symmetric but not antisymmetric and not transitive.

4- The graph K_4 is considered as:

(a) 4-regular complete.
(b) 3-regular not bipartite.
(c) 3-regular bipartite.
(d) None of the previous.

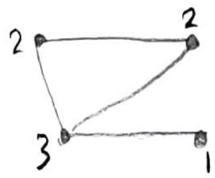
5- $\forall n > 3$, the degree sequence of vertices in $K_{3,n}$ is:

$K_{3,2}$

$2, 2, 2, \underbrace{3, 3}_{2 \text{ times}}$

(a) $\underbrace{n, n, n, \dots, n}_{n \text{ times}}, 3, 3, 3.$
(b) $n, n, n, 3, 3, 3.$
(c) $\underbrace{3, 3, 3, \dots, 3}_{n \text{ times}}.$
(d) $n, n, n, \underbrace{3, 3, 3, \dots, 3}_{n \text{ times}}.$

6- There exists a graph with vertices of degrees



(a) 3, 3, 2, 1.
(b) 6, 4, 3, 2.
(c) 3, 2, 2, 1.
(d) None of the previous.

7- The number of edges in the complementary graph of C_{10} is

(a) 35.
(b) 20.
(c) 25.
(d) 45.

8- In a poset $(P(A), \subseteq)$, where $A = \{0,1,2\}$, then $\{0\}$ and $\{1\}$ is

$4 = 2$

(a) Comparable.
(b) Incomparable.

9- If $M_S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $M_R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then $M_{S \cap R} =$ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

10- If $R = \{(x, y) | y = x^2 - 3\}$, and $S = \{(x, y) | y = 2x\}$, where R and S are defined on $\mathbb{Z}$, then $R \circ S$ is defined as

(a) $\{(x, y) | \ y = (2x^2 - 3)^2\}$.
(b) $\{(x, y) | \ y = 2x^2 - 3\}$.
(c) $\{(x, y) | \ y = 4x^2 + 3\}$.
(d) $\{(x, y) | \ y = 4x^2 - 3\}$.

$y = (2x)^2 - 3$

$y = 4x^2 - 3$

Question II: (3+6 = 9 points)

(A) Let $R = \{(1,2), (2,3), (4,1)\}$ be a relation on a set $\{1, 2, 3, 4\}$. Find

(i) R^2. $R^2 = R \circ R$

$R^2 = \{(1,3),(4,2)\}$

(ii) R^{-1}.

$R^{-1} = \{(2,1),(3,2),(1,4)\}$

(iii) $R \circ R^{-1}$.

$R \circ R^{-1} = \{(1,1),(2,2),(4,4)\}$ -0.5

(iv) $R - R^{-1}$.

$R - R^{-1} = R = \{(1,2),(2,3),(4,1)\}$

$\frac{5.5}{6}$

(B) Let R be the relation defined on the set $\mathbb{Z}$, such that:

$$x, y \in \mathbb{Z}, \quad xRy \Leftrightarrow x + y \text{ is even.}$$

(i) Show that R is an equivalence relation.

$\frac{5.25}{6}$

① $\forall x \in Z$: $a + a = 2a$ then it is even.
$\therefore aRa \Rightarrow R$ is reflexive

② $\forall x, y \in Z$: $xRy \Rightarrow x+y = 2m_1, m_1 \in Z$

$\Rightarrow y = 2m_1 - x$

$\xRightarrow{\text{add } x} y + x = 2m_1 - x + x$

-0.75

$\Rightarrow y + x = 2m_1, m_1 \in Z$, where $m_1 = m$

$\therefore yRx \Rightarrow R$ is symmatric

my goal
yRx
$y+x = 2m, m \in Z$

③ $\forall x, y, c \in Z$: ① $xRy \Rightarrow x+y = 2h_1$
and
② $yRc \Rightarrow y + c = 2h_2$; $h_1, h_2 \in Z$

my goal
xRc
$x+c = 2h, h \in Z$

from ① + ②

$x + 2y + c = 2h_1 + 2h_2$

$\Rightarrow x + c = 2h_1 + 2h_2 - 2y$

$\Rightarrow x + c = 2(h_1 + h_2 - y)$ where $h = h_1 + h_2 - y$

$\Rightarrow x + c = 2h$ it is even

$\therefore xRc \Rightarrow R$ is transitve

from ①, ② and ③ R is equivalnce relation

(ii) Find the equivalence class [2] and [5].

4

$[2] = \{x \in Z : 2Rx \Rightarrow 2+x$ is even $\Rightarrow x$ is even number $\in Z\}$

$= \{ \ldots -6, -4, -2, 0, 2, 4, 6, \ldots \}$

$[5] = \{x \in Z : 5Rx \Rightarrow 5+x$ is even $\Rightarrow x$ is odd number $\in Z\}$

$= \{ \ldots, -7, -5, -3, 1, 3, 5, 7, \ldots \}$

(iii) Is $[7] \cap [10] = \emptyset$ or not. (Justify your answer.)

$7+10 = 17$ not even $\Rightarrow 7 \not R 10$

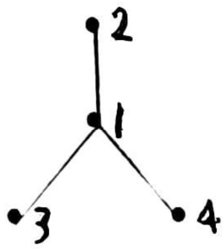
$10+7 = 17$ not even $\Rightarrow 10 \not R 7$

10, 7 incomparple then $[7] \cap [10] = \emptyset$

Question III: (3.25+3=6.25 points)

reflexive + anti + trans

(A) Let T be a partial ordering relation defined on the set $A=\{1,2,3,4\}$ shown in the given Hasse diagram



4.5

(i) List all ordered pairs of T.

$T = \{(1,1), (2,2), (3,3), (4,4), (1,2), (4,1), (4,2), (3,1), (3,2)\}$

2

(ii) Decide whether T is totally ordering on A, or not. (justify your answer)

no it is not totally ordered because $3, 4 \in A$ ~~Not R~~ !

but $3 \not R 4$ and $4 \not R 3$

$\Rightarrow (3,4) \notin T$ and $(4,3) \notin T$

$\frac{3.5}{6}$

(B) Let S be a relation defined on the set $\mathbb{N}$:

$$a, b \in \mathbb{N}, \quad a\,S\,b \Leftrightarrow \frac{a}{b} \text{ is an odd integer.}$$

Show that S is a partial ordering relation on $\mathbb{N}$. $\to \mathbb{Z}^+$

$\frac{1.5}{1.5}$

① $\forall a \in N$: $\frac{a}{a} = 1$ it is odd integer

$\therefore a R a \Rightarrow S$ is reflexive

② $\forall a, b \in N$: assume $a S b \Rightarrow \frac{a}{b}$ is odd

my goal: $\frac{b}{a}$ is odd, $\frac{b}{a} = 2m - 1$, $m \in N$

$\frac{a}{b} = 2m_1 - 1$, $m_1 \in N$

$\frac{0.5}{2.5}$

$\Rightarrow \frac{b\cancel{a}}{a\cancel{b}} = \frac{\cancel{b} \cdot 2(m_1 - 1)}{\cancel{a}}$

$\Rightarrow \frac{b}{a} = 2m_1 - 1$ $\therefore b S a \Rightarrow S$ is antisymm

③ $\forall a, b, c \in N$: ① $a S b \Rightarrow \frac{a}{b} = 2h_1 - 1$

and

② $b S c \Rightarrow \frac{b}{c} = 2h_2 - 1$

my goal: $a S c$, $\frac{a}{c} = 2h - 1$, $h \in N$

from ① × ② $\frac{a\cancel{b}}{c\cancel{b}} = (2h_1 - 1) \cdot (2h_2 - 1)$

$\frac{1.5}{2}$

$\Rightarrow \frac{a}{c} = 2h - 1$ How $= 2(2h_1h_2 - h_1 - h_2) + 1$

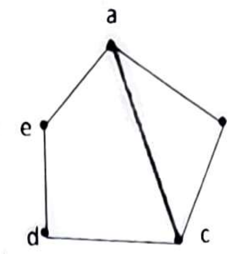
$\therefore a S c \Rightarrow S$ is transitive

$\therefore$ from ①, ② and ③ S is partial order on N

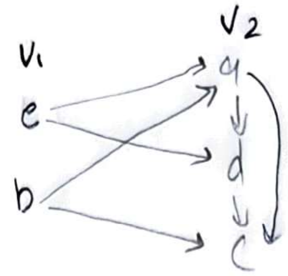
4.5 / 4.5

Question IV: (2.25+2.5=4.75 points)

(A) Answer the following questions about the following graph *G*:



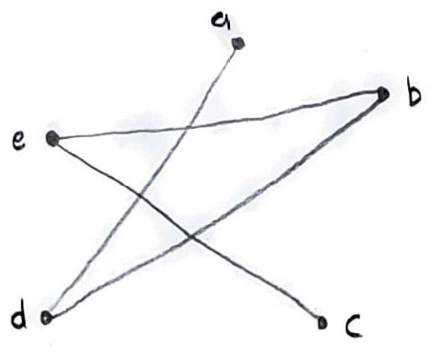
G



2.25 / 2.25 (i) Is the graph *G* bipartite? Justify your answer.

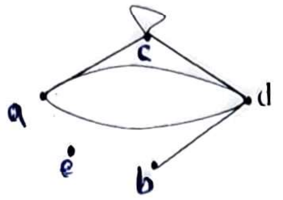
no it is not because we can not sipreate it into 2 group as I draw V2 elements connect to each other

2.25 / 2.25 (ii) Draw the complementary graph of *G*.



(B) For the graph below, find the following:

$\frac{5}{5}$



$\frac{2}{2}$

(i) Find $\deg(a), \deg(c)$.

$\deg(a) = 3$ 1

$\deg(c) = 4$ 1

$\frac{1}{1}$

(ii) Find $N(A)$, *where* $A = \{c, d\}$.

$N(c) = \{c, a, d\}$

$N(d) = \{c, a, b\}$

$N(A) = \{a, c, b, d\}$ 1

$\frac{2}{2}$

(iii) Does the graph have isolated vertex? Pendant vertex? If so, name them.

the isolated vertex is e because $\deg(e) = 0$

the pendant vertex is b because $\deg(b) = 1$

Good Luck