بسم الله الرحمن الرحيم

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STAT 324

First Midterm Exam

First Semester (1431 – 1432 H)

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- Mobile Telephones are not allowed in the classrooms.
- Time allowed is 90 minutes.
- Answer all questions.
- Choose the nearest number to your answer.
- WARNING: Do not copy answers from your neighbors. They have different questions forms.
- For each question, put the code of the correct answer in the following table beneath the question number:

1	2	3	4	5	6	7	8	9	10
D	B	A	B	A	C	A	D	B	D

11	12	13	14	15	16	17	18	19	20
2.25	B	B	D	B	C	D	C	D	A

21	22	23	24	25	26	27	28	29	30
C	A	B	D	B	B	A	A	A	A

▶▶ ▶▶

A town has people with 3 nationalities A, B and C. The town has 70% of A, 20% of B and 10% of C. The record shows that proportion of people committed in a crime are 10% , 4% and 2% respectively for A, B and C. If a person selected randomly from the town

(1)	so the probability that he made a crime?							
	(A)	0.975	(B)	0.875	(C)	2/25	(D)	0.08

(2)	If a person selected randomly from the town made a crime , so the probability that his/her nationality is A?							
	(A)	0.975	(B)	0.875	(C)	2/25	(D)	0.08

(3)	If a person selected randomly from the town made a crime , so the probability that his/her nationality is not C will be							
	(A)	0.975	(B)	0.875	(C)	2/25	(D)	0.08

▶▶ ▶▶ Let Y be a random variable with cumulative distribution function

$$F(y)=\begin{cases} 0 & y<0 \\ 1-e^{-3y} & y\geq 0 \end{cases}.$$

Then

(4)	E(Y) =							
	(A)	0	(B)	0.333	(C)	0.255	(D)	0.777
(5)	P(Y=3)=							
	(A)	0	(B)	0.333	(C)	0.255	(D)	0.777
(6)	P(Y > 0.5) =							
	(A)	0	(B)	0.333	(C)	0.255	(D)	0.777

▶▶ ▶▶

Let X be a discrete random variable with the following cumulative distribution function:

$$F(x)=\begin{cases} 0 \ , & x<3 \\ 0.25 \ , & 3\leq x<4 \\ 0.45 \ , & 4\leq x<7 \\ 0.85 \ , & 7\leq x<9 \\ 1 \ , & x\geq 9 \end{cases}$$

(7)	P(3<X≤7) is							
	(A)	0.6	(B)	0.20	(C)	0.24	(D)	0.16
(8)	P(X=4) is							
	(A)	1.00	(B)	0.80	(C)	0.84	(D)	0.20

▸▸ ▸▸

Let X be a random variable with a probability distribution function

X	-1	1	2	3
f(x)	0.2	0.3	0.1	0.4

Then

(9)	$P(0 < X \leq 2) =$			
	(A) 1.61	(B) 0.4	(C) 0.25	(D) 0.8
(10)	$P(X > 0.5) =$			
	(A) 1.61	(B) 0.4	(C) 0.25	(D) 0.8
(11)	$V(X) =$			
	(A) 1.61	(B) 0.4	(C) 0,25	(D) 0.8

▸▸ ▸▸

A random variable Y has the following distribution:

Y	-1	0	1	2
f(Y=y)	C	2C	3C	4C

(12)	The value of the constant C is:			
	(A) 02	(B) 0.1	(C) 0.25	(D) 2

▸▸ ▸▸

Let X and Y be two independent random variables such that $\mu_x = E(X) = 1$, $Var(X) = \sigma_x^2 = 2$, $\mu_y = E(Y) = -2$, and $Var(Y) = \sigma_y^2 = 1$

(13)	E(X-3Y+1) equals:			
	(A) 11	(B) 8	(C) 38	(D) 40
(14)	Var(X+2) equals:			
	(A) 6	(B) 67	(C) 21	(D) 2
(15)	$E(Y^2)$ equals			
	(A) 1	(B) 5	(C) 2	(D) 0.8
(16)	The lower bound for the probability : P(-4<Y<0) is			
	(A) 0.5	(B) 0.25	(C) 0.75	(D) 1.0

(17)	$E(X-1)^2$			
	(A) 0.2	(B) 0.1	(C) 0.25	(D) 2

▸▸ ▸▸

Suppose that $P(A)=0.4$, $P(B^C)=0.4$, and A and B are independent, then:

(18)	$P(A\cap B)$ equals to							
	(A)	0.00	(B)	0.20	(C)	0.24	(D)	0.16
(19)	$P(AUB)$ equals to							
	(A)	1.00	(B)	0.56	(C)	0.84	(D)	0.76

▸▸ ▸▸

Suppose that $P(A)=0.4$, $P(B^c)=0.4$, and A and B are mutually exclusive events, then:

(20)	$P(A\cap B)$ equals to							
	(A)	0.00	(B)	0.20	(C)	1.00	(D)	0.76
(21)	$P(AUB)$ equals to							
	(A)	0.00	(B)	0.20	(C)	1.00	(D)	0.76

▸▸ ▸▸

If a box contains 3 white balls and 4 black balls, then

(22)	The probability of drawing 2 white balls without replacement is:							
	(A)	1/7	(B)	4/6	(C)	9/49	(D)	1/49
(23)	The probability of getting first and the second balls to be white while the third ball to be black (without replacement) is:							
	(A)	0.23	(B)	0.114	(C)	0.15	(D)	1.00

▸▸ ▸▸

Each of the hundred students taking STAT 324 was asked about his college and whether he smokes or not. Data obtained is as given in the following table:

College	Smoke S	Does not smoke S^c
Computer Science C	7	35
Engineering E	5	18
Other O	8	27

A student is selected at random from this group.

(24)	The probability that the student does not smoke is:							
	(A)	1/2	(B)	7/16	(C)	1/5	(D)	4/5
(25)	The probability that the student smokes given that he is from college of computer science, is:							
	(A)	7/20	(B)	1/6	(C)	1/5	(D)	5/6

(26)	If the student chosen happens to be a smoker, the probability that he is from college of engineering, is:							
	(A)	7/20	(B)	1/4	(C)	1/5	(D)	5/6

▸▸ ▸▸

If $P(A) = 0.5$, $P(B) = 0.4$, and $P(A^C \cup B^C) = 0.6$, then:

(27)	$P(A \cap B)$ equals to							
	(A)	0.40	(B)	0.20	(C)	0.60	(D)	0.90
(28)	$P(A \mid B^c)$ equals to							
	(A)	0.167	(B)	0.00	(C)	0.50	(D)	0.833

▸▸ ▸▸

If $P(A) = 0.8$, $P(B) = 0.5$, and $P(A\ UB) = 0.8$, then:

(29)	The two events A and B are							
	(A)	dependent	(B)	independent	(C)	disjoint	(D)	Mutually exclusive

▸▸ ▸▸

(30)	The number of ways a committee of 4 persons consisting of 2 male and 2 females from 6 men and 3 women is:							
	(A)	45	(B)	18	(C)	189	(D)	24

Good Luck

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$\rightarrow$ 70% A 20% B 10% C

10% Crime (R) 4% 2%

(1) $P(R) = P(A)P(R|A) + P(B)P(R|B) + P(C)P(R|C)$

$= (0.7)(0.1) + (0.2)(0.04) + (0.1)(0.02) = 0.08$

(2) $P(A|R) = \dfrac{P(A)P(R|A)}{P(R)} = \dfrac{(0.7)(0.1)}{0.08} = 0.875$

(3) $1 - P(C|R) = 1 - \dfrac{P(C)P(R|C)}{P(R)} = 1 - \dfrac{(0.1)(0.02)}{0.08} = 0.975$

$\rightarrow$ C.D.F. $F(y) = \begin{cases} 0 & y < 0 \\ 1 - e^{-3y} & y \geq 0 \end{cases}$

$f(y) = \dfrac{d}{dy} F(y) = 3e^{-3y}, \quad y \geq 0$

(4) $E(Y) = \int_0^\infty \underbrace{y}_{u} \cdot \underbrace{3e^{-3y}\,dy}_{dv} = y(-e^{-3y})\Big|_0^\infty - \int_0^\infty -e^{-3y}\,dy$

$= 0 - e^{-3y}/3\Big|_0^\infty = \frac{1}{3} = 0.333$

(5) $P(Y=3) = 0$

(6) $P(Y > 0.5) = \int_{0.5}^\infty 3e^{-3y}\,dy = -e^{-3y}\Big|_{0.5}^\infty = 0 + e^{-1.5}$

$= \dfrac{1}{e^{3/2}} = 0.223$

$\rightarrow$ CDF

$F(x) = \begin{cases} 0 & x < 3 \\ 0.25 & 3 \leq x < 4 \\ 0.45 & 4 \leq x < 7 \\ 0.85 & 7 \leq x < 9 \\ 1 & x \geq 9 \end{cases}$

x	3	4	7	9
$f(x)$	0.25	0.2	0.4	0.15

(7) $P(3 < X \leq 7) = P(X=4) + P(X=7)$

$= 0.2 + 0.4 = 0.6$

(8) $P(X=4) = 0.2$

$\longrightarrow$

x	-1	1	2	3
$f(x)$	0.2	0.3	0.1	0.4

(9) $P(0 < X \leq 2) = P(X=1) + P(X=2)$
$= 0.3 + 0.1 = 0.4$

(10) $P(X > 0.5) = 1 - P(X=-1)$
$= 1 - 0.2 = 0.8$

(11) $Var(X) = E(X^2) - [E(X)]^2$
$= [(1)(0.2) + (1)(0.3) + (4)(0.1) + (9)(0.4)] -$
$[(-1)(0.2) + (1)(0.3) + (2)(0.1) + (3)(0.4)]^2$
$= 4.5 - (1.5)^2 = 2.25$

$\longrightarrow$

Y	-1	0	1	2
$P(Y=y)$	c	$2c$	$3c$	$4c$

(12) $\sum f(y) = 1 \qquad 10c = 1 \qquad c = 0.1$

$\longrightarrow \quad \mu_X = 1, \quad Var(X) = 2, \quad \mu_Y = -2, \quad Var(Y) = 1$

(13) $E(X - 3Y + 1) = E(X) - 3E(Y) + 1 = 8$

(14) $Var(X+2) = Var(X) = 2$

(15) $E(Y^2) = Var(Y) + [E(Y)]^2$
$= 1 + (-2)^2 = 5$

(16) $P(|Y - \mu_Y| < k\sigma_Y) \geq 1 - \frac{1}{k^2}$ Chebyshev's theorem

$P(\mu - k\sigma < Y < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

$P(-4 < Y < 0) \geq 1 - \frac{1}{2^2} \qquad 0.75$

(17) $E((X-1)^2) = Var(X-1) + [E(X-1)]^2$
$= Var(X) + [E(X) - 1]^2 = 2 + 0 = 2$

→ $P(A) = 0.4$, $P(B^c) = 0.4$, A, B are independent

(18) $P(A \cap B) = P(A)\,P(B) = (0.4)(0.6) = 0.24$

(19) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.6 - 0.24 = 0.76$

→ $P(A) = 0.4$, $P(B^c) = 0.4$; A and B are disjoint (mutually exclusive)

(20) $P(A \cap B) = 0$

(21) $P(A \cup B) = P(A) + P(B) = 1$

→ Box contains 3 white and 4 black balls

3 W	4 B
7	

(22) $P(W_1 W_2) = P(W_1)\,P(W_2 | W_1)$

$= \frac{3}{7} * \frac{2}{6} = \frac{1}{7}$

(23) $P(W_1 W_2 B_3) = P(W_1 W_2)\,P(B_3 | W_1 W_2)$

$= \frac{1}{7} * \frac{4}{5} = \frac{4}{35} = 0.114$

→

	S	S^c	
C	7	35	42
E	5	18	23
O	8	27	35
	20	80	100

(24) $P(S^c) = \frac{80}{100} = \frac{4}{5}$

(25) $P(S | C) = \frac{P(S \cap C)}{P(C)} = \frac{7/100}{42/100} = \frac{1}{6}$

(26) $P(E | S) = \frac{P(S \cap E)}{P(S)} = \frac{5/100}{20/100} = \frac{1}{4}$

$\rightarrow$ $P(A) = 0.5$, $P(B) = 0.4$

$P(A^c \cup B^c) = 0.6$

(27) $P(A^c \cup B^c) = P((A \cap B)^c) = 1 - P(A \cap B) = 0.6$

$P(A \cap B) = 0.4$

(28) $P(A \mid B^c) = \dfrac{P(A \cap B^c)}{P(B^c)} = \dfrac{P(A) - P(A \cap B)}{1 - P(B)}$

$= \dfrac{0.5 - 0.4}{1 - 0.4} = 0.167$

$\rightarrow$ $P(A) = 0.8$, $P(B) = 0.5$

$P(A \cup B) = 0.8 \Rightarrow P(A \cap B) = 0.5$

(29) A and B are dependent

$P(A \cap B) \neq P(A)\,P(B)$

(30) $\binom{6}{2}\binom{3}{2} = \dfrac{6!}{2!\,4!} * \dfrac{3!}{1!\,2!} = 45$