King Saud University	College of Sciences		Department of Mathematics
First Examination	Math 132	Semester 2 (1445)	Time:1H30

Question 1: (9 marks(3+3+3))

1. Decide whether the following propositions is a tautology or a contradiction or a contingency?
$$(p \vee \neg q) \rightarrow (\neg q \leftrightarrow r).$$
2. Without using truth tables, prove that the following conditional statement is a Tautology:
$$p \rightarrow [(p \wedge q) \vee \neg q].$$
3. Without using truth tables, prove the following logical equivalence:
$$(p \rightarrow q) \wedge (\neg p \rightarrow q) \equiv q.$$

Question 2: (11 marks (2+2+3+4))

1. Let a, b be integers. Use a proof by contraposition to show that if $(a.b)$ is an even number, then a is even or b is even.
2. Let x, y and z be three real numbers such that $2x + y + 3z = 21$. Use a proof by contradiction to show that $x \geq 4$ or $y \geq 7$ or $z \geq 2$.
3. Use mathematical induction to show that:
$$1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}, \quad \text{for each integer } n, \text{ with } n \geq 1.$$
4. Consider the sequence $\{u_n\}_{n=0}^{\infty}$ defined as follows: $\begin{cases} u_1 = 0 \\ u_2 = 1 \\ u_{n+1} = 3u_n - 2u_{n-1} - 1; \quad n \geq 2 \end{cases}$
Use mathematical induction to prove the following statement:
$$u_n = n - 1, \quad \text{for each integer } n, \text{ with } n \geq 1.$$

Question 3: (5 marks (2+3))

1. Consider the two sets $A := \{1, 2, 3, 4, \{1\}, \{2\}, \{1, 2\}, \{1, \{1\}\}, \{\{1\}, \{2\}\}\}$. Determine whether each of the following seven statements is true or false. (Justify your answer).
(i) S_1: "$\{1, 3\} \in A$". **(ii)** S_2: "$\{1, 4\} \subseteq A$".
(iii) S_3: "$\{2, \{2\}\} \subseteq A$". **(iv)** S_4: "$A \cap \{1, 2, \{\{1\}, \{2\}\}\} = \{1, 2\}$".
2. Consider the following three sets $C := \{a, b, c\}, D := \{1, 2, a\}$, and
$E := \{(a, 1), (1, a), (b, b), (2, 2), (c, 2), (c, a)\}$. Find the following sets:
(i) $(C \cap D) \times C$. **(ii)** $E \setminus (C \times D)$. **(iii)** $\emptyset \times E$.