

King Saud University
College of Sciences
Department of Mathematics
Math-244 (Linear Algebra); Mid-term Exam; Semester 1 (1442)
Max. Marks: 30 **Time: 2 hours**

Note: Attempt all the five questions!

Question 1: [Marks: 2+3]

- a) Let $A = \begin{bmatrix} 1 & -1 & 4 & 5 \\ -2 & 1 & -11 & -8 \\ -1 & 2 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 1 & 4 & 1 \\ 1 & 0 & 8 & 6 \end{bmatrix}$. Then show that the matrices A and B are row equivalent to each other.
- b) Give any two matrices A and B that satisfy:
 $\text{trace}(A+B) = \text{trace}(A) + \text{trace}(B)$ and $\text{trace}(AB) \neq \text{trace}(A)\text{trace}(B)$.

Question 2: [Marks: 2+3]

- a) Let $A, B \in M_2(\mathbb{R})$ with $|A| = 3$ and $|B| = 6$. Then evaluate $||A|A^t B^2 \text{adj}(A^2)|$.
- b) Let $A = \begin{bmatrix} 1 & 0 & \delta \\ 2 & 1 & 2 + \delta \\ 2 & 3 & \delta^2 \end{bmatrix}$. Find the values of δ if the matrix A is not invertible.

Question 3: [Marks: 2+4]

- a) Find the values of x and y if $A = \begin{bmatrix} - & 2 & - \\ - & x & - \\ - & y & - \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ - & - & - \end{bmatrix}$.
- b) Find the value/s of α such that the following linear system:
- $$\begin{array}{rrcr} x & +2y & - & z & = & 2 \\ x & -2y & + & 3z & = & 1 \\ x & +2y & - & (\alpha^2 - 3)z & = & \alpha \end{array}$$

has:

- (i) no solution (ii) unique solution (iii) infinitely many solutions.

Question 4: [Marks: 2+3+3]

- a) Let $S = \{(1,1,1,0), (1,2,3,1), (2,0,1,1)\}$ generates the subspace F of Euclidean space \mathbb{R}^4 . Show that $(1, 1, 1, 1) \notin F$.
- b) Let $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $C = \{(1,1,1), (1,2,2), (1,1,2)\}$ be bases of the Euclidean space \mathbb{R}^3 and $[v]_B = [1 \ 2 \ 3]^T$. Find the transition matrix ${}_C P_B$ and $[v]_C$.
- c) Let $A^T = \begin{bmatrix} 1 & 2 & 2 & 0 & 0 \\ 2 & 4 & 5 & 2 & 2 \\ 1 & 2 & 3 & 2 & 2 \\ 3 & 6 & 4 & -3 & -4 \end{bmatrix}$. Then find:
- (i) a basis of $col(A)$ (ii) $rank(A)$ (iii) $nullity(A)$.

Question 5: [Marks: 2+1+3]

Let $S = \{v_1 = (1, -1, 0, 1), v_2 = (1, 1, 1, 0), v_3 = (0, 1, 1, 1)\}$ generates the subspace W of the Euclidean space \mathbb{R}^4 . Then:

- a) Show that S is a basis of W .
- b) Find the angle θ between the vectors v_1 and v_2 .
- c) Apply the Gram-Schmidt process on S to obtain an orthonormal basis of W .

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