ME-392: Mechanics Fall 1427/1428 (27-2)

HW 3 Solution

Problem 3.15

A precast-concrete wall section is temporarily held by cables as shown. Knowing that the tension in cable BC is 900 Ib, determine the moment about the origin of coordinates O of the force exerted on the wall section at C.

Solution

The moment about point *O* is the cross product of the position vector $\mathbf{r}_{OC}$ and the tension $\mathbf{T}_{CB}$

$$\vec{M}_O = \vec{r}_{OC} \times \vec{T}$$

The position vector is obtained by the difference between the coordinates of the two points:

$$\vec{r}_{OC} = \vec{r}_C - \vec{r}_O$$

$$\vec{r}_{OC} \quad \cdots \quad O(0,0,0); C(0,6,0)$$

$$\therefore \qquad \vec{r}_{OC} = 0\hat{i} + 6\hat{j} + 0\hat{k}$$

Direction of $\mathbf{T}_{CB}$ is same as the direction of $\mathbf{r}_{CB}$

$$\vec{r}_{CB} \cdots \quad C(0,6,0); B(12,0,12);$$

$$\vec{r}_{CB} = 12\hat{i} - 6\hat{j} + 12\hat{k}$$

$$|\vec{r}_{CB}| = \sqrt{(12)^2 + (-6)^2 + (12)^2} = 18$$

$$\hat{e}_{CB} = \frac{12}{18}\hat{i} - \frac{6}{18}\hat{j} + \frac{12}{18}\hat{k} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

$$\vec{T}_{CB} = 900(\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k})$$

$$\vec{T}_{CB} = 600\hat{i} - 300\hat{j} + 600\hat{k}$$

$$\vec{M}_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 6 & 0 \\ 600 & -300 & 600 \end{vmatrix}$$

$$\vec{M}_O = ((6\times600) - 0)\hat{i} - (0)\hat{j} + (0 - (6\times600))\hat{k}$$

$$\vec{M}_O = (3600\hat{i} - 3600)\hat{k} \quad \text{lb}\cdot\text{ft}$$

or

$$\vec{M}_O = 6\hat{i} \times (600\hat{i} - 300\hat{j} + 600\hat{k}) = (3600\hat{i} - 3600\hat{k}) \quad \text{lb}\cdot\text{ft}$$

Problem 3.18

A force **Q** of magnitude 450 N is applied at *C* as shown. Determine the moment of **Q** about (a) the origin of coordinates *O*. (b) point *D*?

Solution

(a) The moment of **Q** about the origin *O*. We first get the position vector $\mathbf{r}_{OC}$ by taking the difference between the coordinates of the two points (in meters)

$$\vec{r}_{OC} \cdots O(0,0,0); \quad C(0.225,0.1,0)$$

$$\vec{r}_{OC} = 0.225\hat{i} + 0.1\hat{j} + 0\hat{k}$$

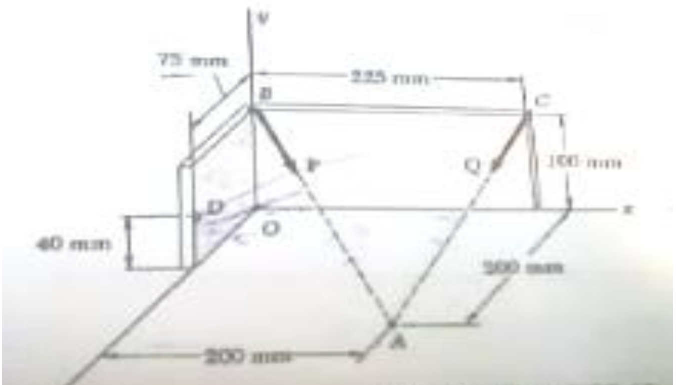


Fig. P3.17 and P3.18

The direction of **Q** is along the direction of the position vector $\mathbf{r}_{CA}$.

$$\vec{r}_{CA} \cdots C(0.225,0.1,0); A(0.2,0,0.2)$$

$$\vec{r}_{CA} = -0.025\hat{i} - 0.1\hat{j} + 0.2\hat{k}$$

$$|\vec{r}_{CA}| = \sqrt{(-0.025)^2 + (-0.1)^2 + (+0.2)^2} = 0.225$$

$$\hat{e}_{CA} = -\frac{0.025}{0.225}\hat{i} - \frac{0.1}{0.225}\hat{j} + \frac{0.2}{0.225}\hat{k} = -\frac{1}{9}\hat{i} - \frac{4}{9}\hat{j} + \frac{8}{9}\hat{k}$$

$$\vec{Q} = Q \ \ \hat{e}_{CA} = 450 \ \ (-\frac{1}{9}\hat{i} - \frac{4}{9}\hat{j} + \frac{8}{9}\hat{k}) = (-50\hat{i} - 200\hat{j} + 400\hat{k}) \text{ N.}$$

Therefore, the moment about *O* can be obtained

$$\vec{\mathrm{M}}_O = \vec{r}_{OC} \times \vec{Q}$$

Or $$\vec{M}_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.225 & 0.1 & 0 \\ -50 & -200 & 400 \end{vmatrix}$$

$$\vec{M}_o = (40\hat{i} - 90\hat{i} - 40\hat{k}) \quad \text{N·m}$$

(b) The moment of **Q** point *D*. We first get the position vector $\mathbf{r}_{DA}$

$$\vec{r}_{DA} \cdots D(0,.04,0.075); A(0.2,0,0.2)$$

$$\vec{r}_{oc} = 0.2\hat{i} - 0.04\hat{j} + 0.125\hat{k}$$

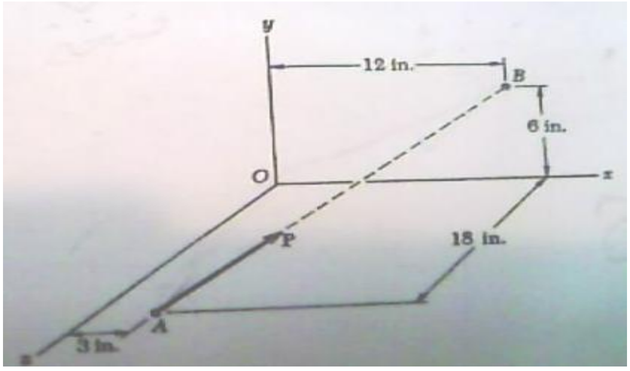
Therefore $\vec{\mathrm{M}}_o = \vec{r}_{CA} \times \vec{Q}$

$$\vec{M}_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.2 & -0.04 & 0.125 \\ -50 & -200 & 400 \end{vmatrix}$$

$$\vec{M}_o = (9\hat{i} - 86.25\hat{i} - 42\hat{k}) \quad \text{N·m}$$

Problem 3.19

The line of action of the force **P** magnitude 420Ib passes through the two points *A* and *B* as shown. Compute the moment of **P** about *O* using the position vector (a) of point *A*, (b) of point *B*.



Solution

First we get the direction of **P** which is along the direction of $\mathbf{r}_{AB}$.

$$\vec{r}_{AB} \cdots A(3,0,18); \quad B(12,6,0)$$

$$\vec{r}_{AB} = 9\hat{i} + 6\hat{j} - 18\hat{k}$$

$$|\vec{r}_{AB}| = \sqrt{(9)^2 + (6)^2 + (-18)^2} = 21$$

$$\hat{e}_{AB} = \frac{9}{21}\hat{i} + \frac{6}{21}\hat{j} - \frac{18}{21}\hat{k} = \frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k}$$

$$\vec{P} = 420(\frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{6}{7}\hat{k})$$

$$\vec{P} = 180\hat{i} + 120\hat{j} - 360\hat{k}$$

Then, we get the two position vectors

a) $\mathbf{r}_{OA}$

$$\vec{\mathrm{M}}_o = \vec{r}_{OA} \times \vec{P}$$

$$\vec{r}_{OA} \cdots O(0,0,0); \quad A(3,0,18)$$

$$\vec{r}_{OA} = 3\hat{i} + 18\hat{k}$$

$$\therefore \quad \vec{M}_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 18 \\ 180 & 120 & -360 \end{vmatrix}$$

$$\vec{M}_o = (-2160\hat{i} + 4320\hat{i} + 360\hat{k}) \text{ Ib·ft}$$

b) $\mathbf{r}_{OB}$

$$\vec{\mathrm{M}}_o = \vec{r}_{OB} \times \vec{P}$$

$$\vec{r}_{OA} \cdots A(0,0,0); B(12,6,0)$$

$$\vec{r}_{OA} = 12\hat{i} + 6\hat{j}$$

Therefore

$$\vec{M}_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 12 & 6 & 0 \\ 180 & 120 & -360 \end{vmatrix}$$

$$\vec{M}_o = (-2160\hat{i} + 4320\hat{i} + 360\hat{k}) \text{ Ib·ft}$$

Notice that we have the same answer from (a) and (b)

Problem 3.28

Determine the angle formed by cables *AD* and *AB*.

Solution

To get the angle between the two lines, we need to get the two position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AD}$, then use the dot product between them:

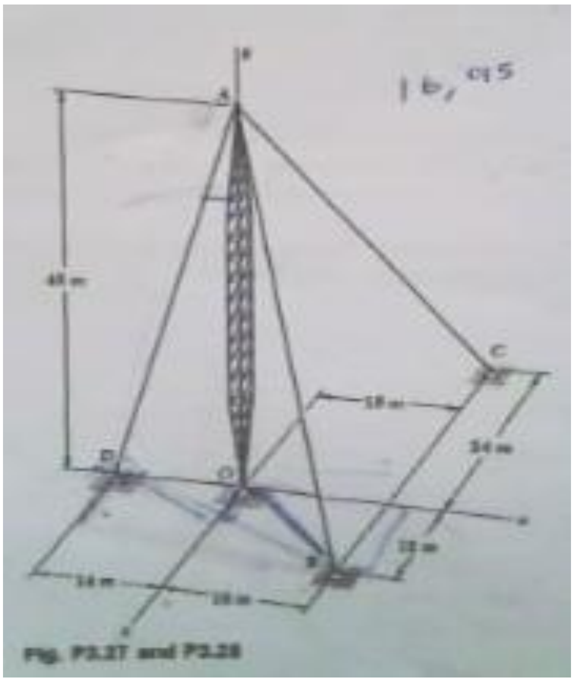


Fig. P3.27 and P3.28

$$\vec{r}_{AD}\cdot\vec{r}_{AB}=\left|\vec{r}_{AD}\right|\cdot\left|\vec{r}_{AB}\right|\cdot\cos\theta$$

So

$\vec{r}_{AD}\quad\cdots\quad A(0,-48,0);\quad D(-14,0,0)$

$\vec{r}_{AD}=-14\hat{i}+48\hat{j}$

$\left|\vec{r}_{AD}\right|=\sqrt{(-14)^2+(48)^2}=50$

$\vec{r}_{AB}\quad\cdots\quad A(0,-48,0);\quad B(16,0,12)$

$\vec{r}_{AD}=16\hat{i}+48\hat{j}+15\hat{k}$

$\left|\vec{r}_{AD}\right|=\sqrt{(16)^2+(48)^2+(12)^2}=52$

$\vec{r}_{AD}\cdot\vec{r}_{AB}=\left|\vec{r}_{AD}\right|\cdot\left|\vec{r}_{AB}\right|\cdot\cos\theta$

$(-14\hat{i}+48\hat{j})\cdot(16\hat{i}+48\hat{j}+15\hat{k})=50\times52\times\cos\theta$

$(-14\times16)(\hat{i}\times\hat{i})+(48\times48)(\hat{j}\times\hat{j})=2600\cos\theta$

$2080=2600\cos\theta$

Therefore $\quad\theta=\cos^{-1}\dfrac{2080}{2600}=36.9^\circ$

Problem 3.32

Knowing that the tension in cable *BD* is 180 Ib, determine (a) the angle between cable *BD* and the boom AB, (b) the projection on *AB* of the force exerted by cable *BD* at point *B.*

Solution

a)Note that we get the angle by the dot product of the two vectors originating from point *B* to *A* and *D.*

So,

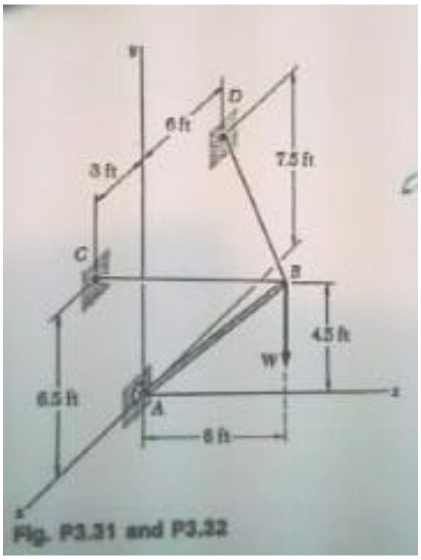


Fig. P3.31 and P3.32

$\vec{r}_{BA}\quad\cdots\quad B(6,4.5,0);\quad A(0,0,0)$

$\vec{r}_{BA}=-6\hat{i}-4.5\hat{j};\quad\left|\vec{r}_{AB}\right|=\sqrt{(-6)^2+(-4.5)^2}=7.5$

$\vec{r}_{BD}\quad\cdots\quad B(6,4.5,0);\quad D(0,7.5,-6)$

$\vec{r}_{BD}=-6\hat{i}+3\hat{j}-6\hat{k}$

$\left|\vec{r}_{BD}\right|=\sqrt{(6)^2+(-3)^2+(6)^2}=9$

$\vec{r}_{BD}\cdot\vec{r}_B A=\left|\vec{r}_{BD}\right|\cdot\left|\vec{r}_{BA}\right|\cdot\cos\theta$

$(-6\hat{i}-4.5\hat{j}+0\hat{k})\cdot(-6\hat{i}+3\hat{j}+6\hat{k})=7.5\times9\times\cos\theta$

$(-6\times-6)+(-4.5\times3)+0=67.5\cos\theta$

$36+(-13.5)=67.5\cos\theta$

$\theta=\cos^{-1}\dfrac{22.5}{67.5}=70.5^\circ$

b) Let the projection on *AB* of T_{BD} at point *B* be P_{BD}

$$P_{AB} = T_{BD}\cos\theta$$

$$T_{AB} = 180\times\cos 70.5^\circ = 180\times 0.334 = 60Ib$$

<u>Problem 3.44</u>

The rectangular plate *ABCD* is held by hinges along its edge by the wire *BE*. Knowing that the tension in wire 546 N, determine the moment about *AD* of the force exerted by the wire at point *B*.

<u>Solution</u>

We first get the moment about point *A*.

$$\vec{r}_{AB} = .45\hat{i}$$

$$\vec{r}_{BE} \quad \cdots \quad B(0.45,0,0);\quad E(0,0.225,0.15)$$

$$\vec{r}_{BE} = -.45\hat{i} + .225\hat{j} + .15\hat{k}$$

$$\left|\vec{r}_{BE}\right| = \sqrt{(-.45)^2 + (.225)^2 + (.15)^2} = .525$$

$$\hat{e}_{BE} = \frac{1}{21}(-18\hat{i} + 9\hat{j} + 6\hat{k})$$

$$\vec{T}_{BE} = T_{BE}\quad \hat{e}_{BE} = \frac{546}{21}(-18\hat{i} + 9\hat{j} + 6\hat{k})$$

$$\vec{M}_A = \vec{r}_{AB}\times\vec{T}_{BE}$$

$$\vec{M}_A = .45\hat{i}\times\frac{546}{21}(-18\hat{i} + 9\hat{j} + 6\hat{k})$$

$$= \frac{9}{20}\left(\frac{546}{21}\right)(9\hat{k} - 6\hat{j}) = \frac{27}{10}(26)(-2\hat{j} + 3\hat{k})\quad \text{N}\cdot\text{m}$$

Then we get the moment about the line *AD* by getting the dot product of M_A and the unit vector $\mathbf{e}_{AD}$

$$\vec{r}_{AD} = -.125\hat{i} + .300\hat{k}$$

$$\hat{e}_{AD} = \frac{1}{13}(-5\hat{j} + 12\hat{k})$$

$$M_{AD} = \hat{e}_{AD}\cdot\vec{M}_A$$

$$\vec{M}_A = \frac{1}{13}(-5\hat{j} + 12\hat{k})\cdot\frac{27}{10}(26)(-2\hat{j} + 3\hat{k})$$

$$= 2.7(10 + 36) = 124.2\quad \text{N}\cdot\text{m}$$