KING SAUD UNIVRESITY
COLLEGE OF ENGINEERING
MECHANICAL ENGINEERING DEPARTMENT

ME 363: Mechanics of Machinery

Spring 1431/1432 (31-2)

Cams

Cam Nomenclature[1]

Figure1 illustrates some cam nomenclature:

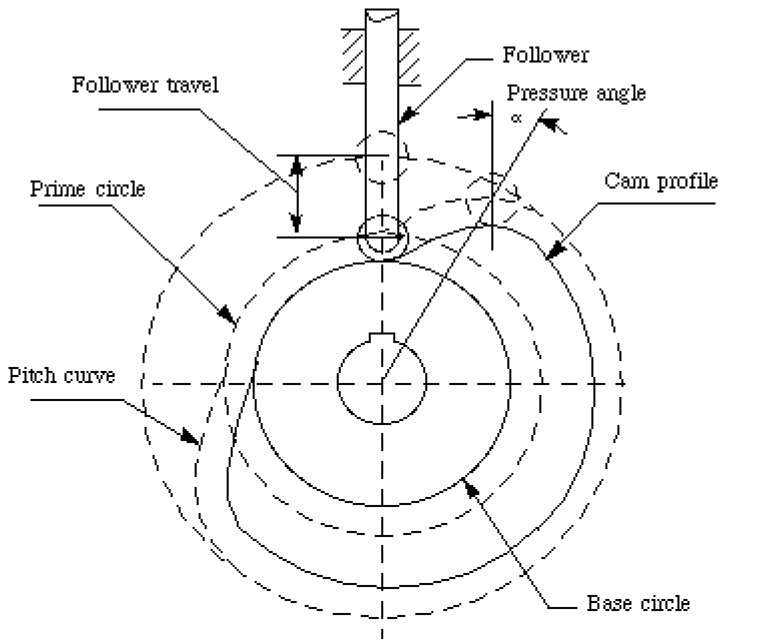


Figure 1 Cam nomenclature

- **Trace point**: A theoretical point on the follower, corresponding to the point of a fictitious *knife-edge follower*. It is used to generate the *pitch curve*. In the case of a *roller follower*, the trace point is at the center of the roller.
- **Pitch curve**: The path generated by the trace point at the follower is rotated about a stationary cam.
- **Working curve**: The working surface of a cam in contact with the follower. For the *knife-edge follower* of the plate cam, the *pitch curve* and the *working curves* coincide. In a *close or grooved cam* there is an *inner profile* and an *outer working curve*.
- **Pitch circle**: A circle from the cam center through the pitch point. The pitch circle radius is used to calculate a cam of minimum size for a given *pressure angle*.
- **Prime circle** (**reference circle**): The smallest circle from the cam center through the pitch curve.

[1] This section is adopted from **Introduction to Mechanisms,** Yi Zhang *et.al*.

- **Base circle**: The smallest circle from the cam center through the cam profile curve.
- **Stroke** or **throw**:The greatest distance or angle through which the follower moves or rotates.
- **Follower displacement**: The position of the follower from a specific zero or rest position (usually its the position when the *f* ollower contacts with the *base circle* of the cam) in relation to time or the rotary angle of the cam.
- **Pressure angle**: The angle at any point between the normal to the pitch curve and the instantaneous direction of the follower motion. This angle is important in cam design because it represents the steepness of the cam profile.

Motion events

When the cam turns through one motion cycle, the follower executes a series of events consisting of rises, dwells and returns. **Rise** is the motion of the follower away from the cam center, **dwell** is the motion during which the follower is at rest; and **return** is the motion of the follower toward the cam center.

There are many follower motions that can be used for the rises and the returns. Here, we describe a number of basic curves.

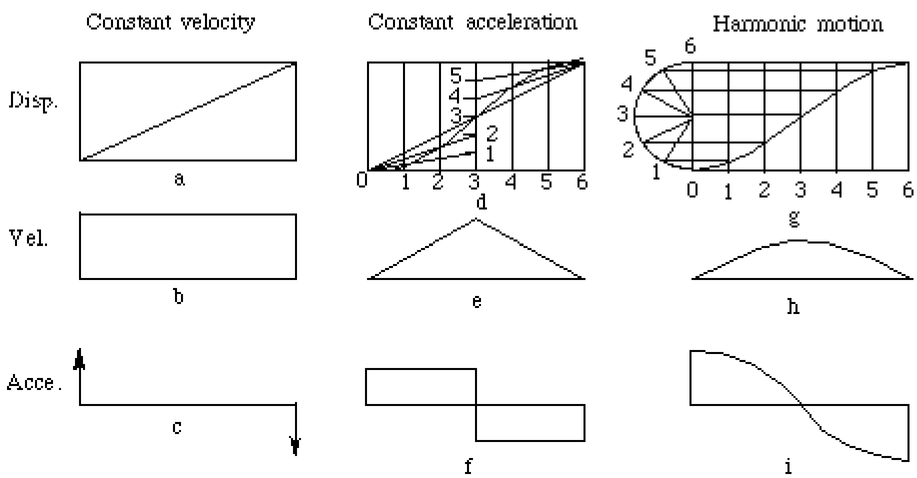


Figure 2: Motion events

Notation

Φ: The rotary angle of the cam, measured from the beginning of the motion event;
β: The range of the rotary angle corresponding to the motion event;
h : The stoke of the motion event of the follower;
S : Displacement of the follower;
V : Velocity of the follower;
A : Acceleration of the follower.

1 Constant Velocity Motion

If the motion of the follower were a straight line, Figure 2 a,b,c, it would have equal displacements in equal units of time, *i.e.*, uniform velocity from the beginning to the end of the stroke, as shown in b. The acceleration, except at the end of the stroke would be zero, as shown in c. The diagrams show abrupt changes of velocity, which result in large forces at the beginning and the end of the stroke. These forces are undesirable, especially when the cam rotates at high velocity. The *constant velocity motion* is therefore only of theoretical interest.

$$S(\varphi) = h\frac{\varphi}{\beta}$$

$$V(\varphi) = \frac{h}{\beta}$$

$$A(\varphi) = 0$$

2 Constant Acceleration Motion

Constant acceleration motion is shown in Figure 2 d, e, f. As indicated in e, the velocity increases at a uniform rate during the first half of the motion and decreases at a uniform rate during the second half of the motion. The acceleration is constant and positive throughout the first half of the motion, as shown in f, and is constant and negative throughout the second half. This type of motion gives the follower the smallest value of maximum acceleration along the path of motion. In high-speed machinery this is particularly important because of the forces that are required to produce the accelerations.

When

$0 \le \varphi \le \frac{\beta}{2}$,

$$S(\varphi) = 2h\frac{\varphi^2}{\beta^2}$$

$$V(\varphi) = \frac{4h}{\beta^2}\varphi$$

$$A(\varphi) = \frac{4h}{\beta^2}$$

When

$\frac{\beta}{2} \le \varphi \le \beta$,

$$S(\varphi) = h - \frac{2h}{\beta^2}(\beta - \varphi)^2$$

$$V(\varphi) = \frac{4h}{\beta}\left(1 - \frac{\varphi}{\beta}\right)$$

$$A(\varphi) = -\frac{4h}{\beta^2}$$

3 Harmonic Motion

A cam mechanism with the basic curve like g in Figure 2 will impart *simple harmonic motion* to the follower. The velocity diagram at h indicates smooth action. The acceleration, as shown at i, is maximum at the initial position, zero at the mid-position, and negative maximum at the final position.

$$S(\varphi) = \frac{h}{2}\left(1 - \cos\frac{\pi\varphi}{\beta}\right)$$

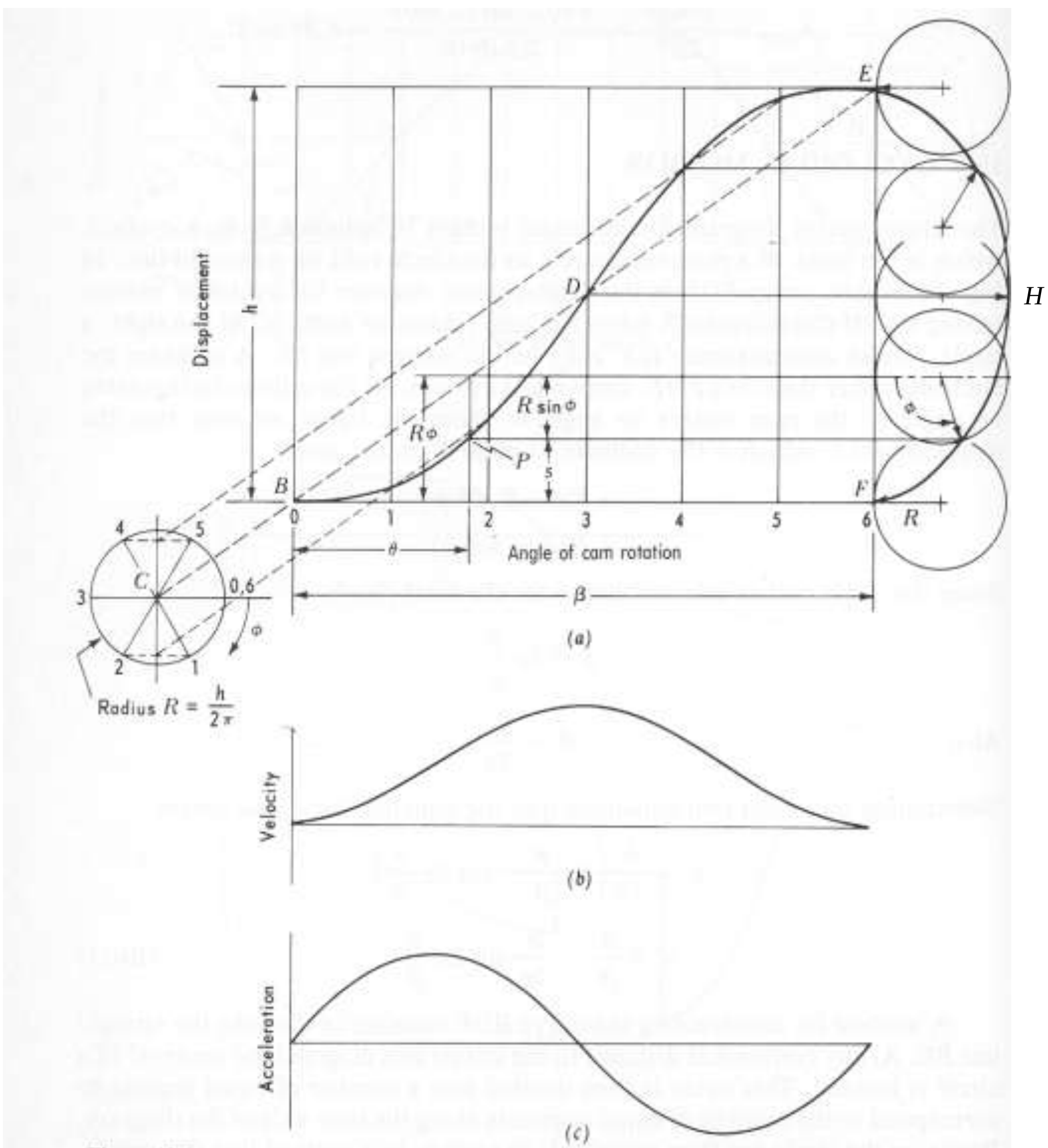
$$V(\varphi) = \frac{h\pi}{2\beta}\sin\frac{\pi\varphi}{\beta}$$

$$A(\varphi) = \frac{h\pi^2}{2\beta^2}\cos\frac{\pi\varphi}{\beta}$$

4 Cycloidal Motion[2]

Displacement diagrams for parabolic and simple harmonic types of follower motion were discussed before. In this class we will introduce the displacement diagram for cycloidal type of motion.

A cycloid is the locus of a point on the perimeter (circumference) of a circle rolling on a straight line. In Figure 3, the circle with radius $R = (h/2\pi)$ rolls on the straight line *FE*, with *h* being the maximum rise of the follower. A point on the circle describes the cycloidal curve *FHE*.



[2] Figures here onward taken from **Kinematics and Dynamics of Machines**, Martin

Figure 3: Cycloid Motion events

Graphical Construction

With reference to the figure above, the displacement diagram for follower cycloidal motion can be constructed as follows:

1. Divide the horizontal time axis into equal even number of divisions.
2. Draw the diagonal straight line *EB* and extend it to the lower left of the diagram
3. Draw a circle of a radius $R = (h/2\pi)$ at a convenient distance on the extension of the diagonal.
4. Divide the circle into a number of divisions similar to that in step 1 above, starting from +ve x-axis and moving clockwise.
5. Project points from the circle circumference on its vertical diameter, and then on a direction parallel to the diagonal until each line intersects with vertical line from corresponding point on displacement horizontal axis.

Equations of Motion

From the graph, we see that the displacement s is given by

$$s = R\phi - R\sin\phi$$

$$= R(\phi - \sin\phi)$$

Since the circle makes a full revolution for a total rise of h

$$\phi = 2\pi\frac{\theta}{\beta}$$

Also, from step 3 above,

$$R = \frac{h}{2\pi}$$

Therefore,

$$s = \frac{h}{2\pi}\left(2\pi\frac{\theta}{\beta} - \sin 2\pi\frac{\theta}{\beta}\right)$$

$$s = h\frac{\theta}{\beta} - \frac{h}{2\pi}\sin 2\pi\frac{\theta}{\beta}$$

For velocity we differentiate with respect to time:

$$v = \frac{\omega}{\beta}h\left(1 - \cos 2\pi\frac{\theta}{\beta}\right)$$

For acceleration we differentiate again

$$a = \frac{2\pi\omega^2}{\beta^2}h\sin 2\pi\frac{\theta}{\beta}$$

The jerk is the third derivative with respect to time and it is given by

$$j = 4\pi^2\frac{\omega^3}{\beta^3}h\cos 2\pi\frac{\theta}{\beta}$$

Note:

In cycloidal motion, acceleration curve starts and ends the rise with zero value, which makes continuity of acceleration with preceding and following dwell or cycloidal motion. This makes the cycloidal motion superior to other types of motion studied.

Cam Design

From the displacement diagram, and with the proper follower type chosen, the cam profile can be constructed to provide the desired motion output. The shape of the profile depends on:

- The size of the cam (decided mainly by the base circle)
- The size, shape and path of the follower

The principle of inversion is used in the construction; that is, holding the cam fixed and rotating the follower and the frame in a direction opposite to cam rotation.

Typical procedure for construction of the cam profile: See Figure 4 below, for the case of knife-edge in-line follower:

1. Draw a circle with radius of the base circle *(r_b)*. For knife edge follower (as above, it meets the follower tip, for roller follower, the roller radius must be added to it).
2. Divide the displacement diagram and the cam into equal number of divisions (12 in this case).
3. The ordinate (value of rise) of each point on the displacement diagram is laid off along the follower.
4. Since the cam motion (here) is clockwise, the follower is rotated counter clockwise, and at the same time pushed upward at each division with the corresponding rise value. This is done for all points to complete full rotation.
5. A smooth curve tangential to all the points is drawn, which is the required cam profile.

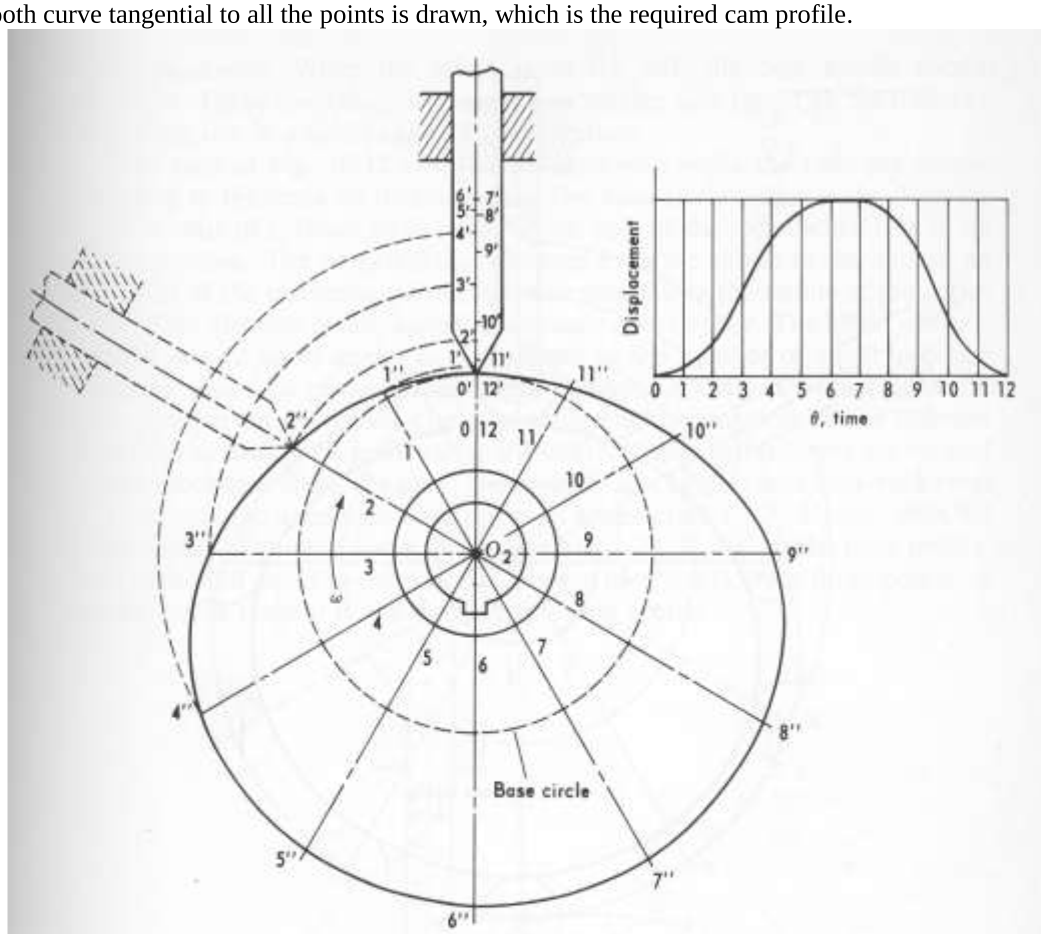


Figure 4: Cam Profile for knife–edge follower

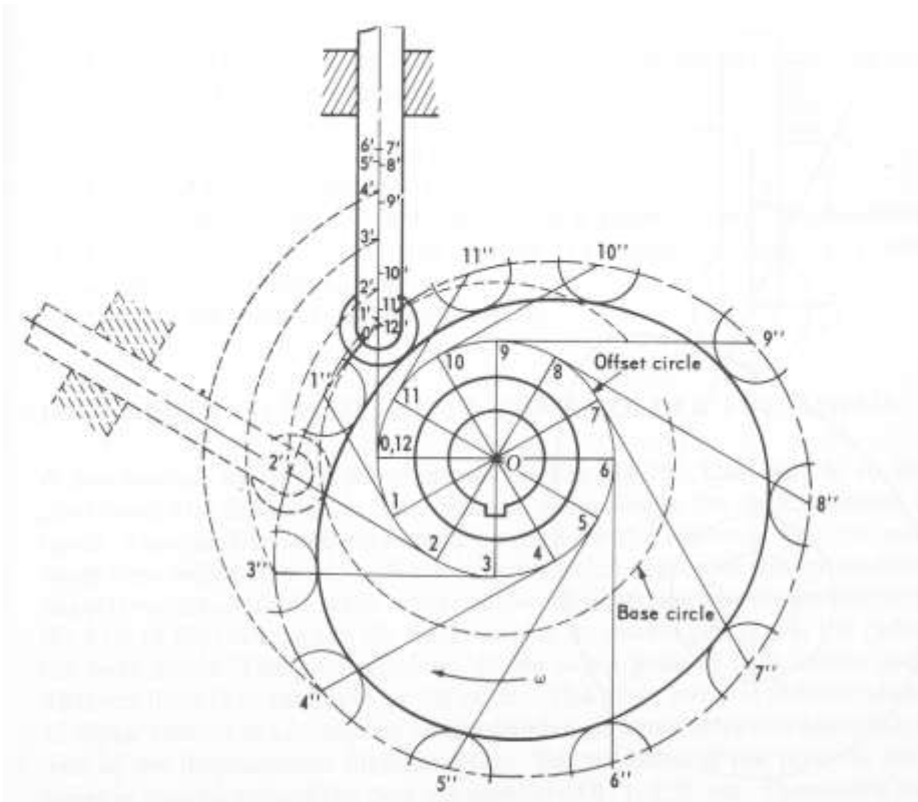


Figure 5: Cam Profile for Roller follower with offset.

For a roller follower with off-set (see Figure 5)

- Draw the prime circle with the radius equal to the distance from the center of the cam to the center of the roller.
- Draw the off-set circle from cam center with radius equal to off-set amount.
- Divide off-set circle to the same no of divisions as the displacement diagram.
- Draw a tangent at each position on the off-set circle to represent the position of the follower centerline.

Example: Draw the profile of a cam operating a roller reciprocating follower and with the following data: Minimum radius of cam = 25 mm; Lift = 30 mm; Roller diameter = 15 mm. The cam lifts the follower for 120° with Simple Harmonic Motion (SHM) followed by a dwell period of 30°. Then the follower lowers down during 150° of the cam rotation with uniform acceleration and deceleration followed by a dwell period. If the cam rotates at a uniform speed of 150 rpm, draw the cam profile and calculate the maximum velocity and acceleration of the follower during the descent period. Assume that the cam is to rotate in the clockwise direction.

Solution:

Lift: $h = 30$ mm

Rotation Speed $N = 150$ rpm

Minimum radius of cam $r_c = 25$ mm

Roller radius $r_r = 7.5$ mm

Draw the displacement diagram of the follower as shown in Fig. (1)

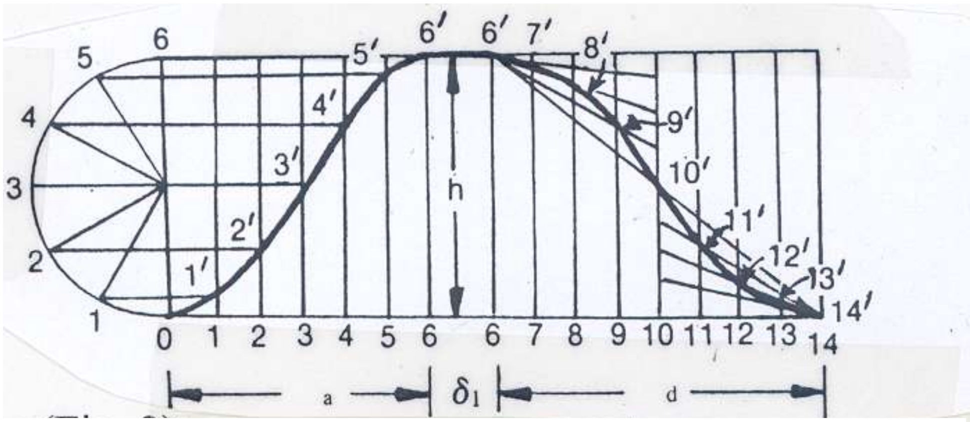


Figure 6

Angles:

Rise (ascend) $\varphi_a = 120°$

Dwell 1 $\delta_1 = 30°$

Return (descend) $\varphi_d = 150°$

Dwell 2 $\delta_2 = 60°$

Construct the cam profile as described below (Fig. 2):

1. Draw a circle with radius $(r_c + r_r)$.
2. From the vertical position, mark angles φ_a, δ_1, φ_d *and* δ_2 in the counter-clockwise direction.
3. Divide the angles φ_a and φ_d into same number of parts as in the displacement diagram, In this case, φ_a has been divided into 6 equal parts whereas φ_d into 8 equal parts.
4. On the radial lines produced, mark the distances from the displacement diagram.
5. Draw a series of arcs of radii equal to r_r as shown in the diagram from the points 1', 2', 3', etc.
6. Draw smooth curve tangential to all the arcs, which is the required cam profile.

During the descent period, the acceleration and the deceleration are uniform. Therefore, the maximum velocity is at the end of the acceleration period, or at midway of the return period, that is: $\theta = \varphi_d/2$. From which:

$$v_{\max} = \frac{4h\omega}{\phi_d^2}\theta = \frac{2h\omega}{\phi_d}$$

Substituting

$$v_{\max} = \frac{2 \times 30 \times (150 \times 2\pi / 60)}{150 \times (\pi / 180)} = 360 \text{ mm/s}$$

Acceleration is uniform and is directly given by the relation.

$$a = \frac{4h\omega^2}{\phi_d^2} = \frac{4 \times 30 \times (150 \times 2\pi / 60)^2}{(150 \times \pi / 180)^2} = 4320 \text{ mm/s}^2 = 4.32 \text{ m/s}^2$$

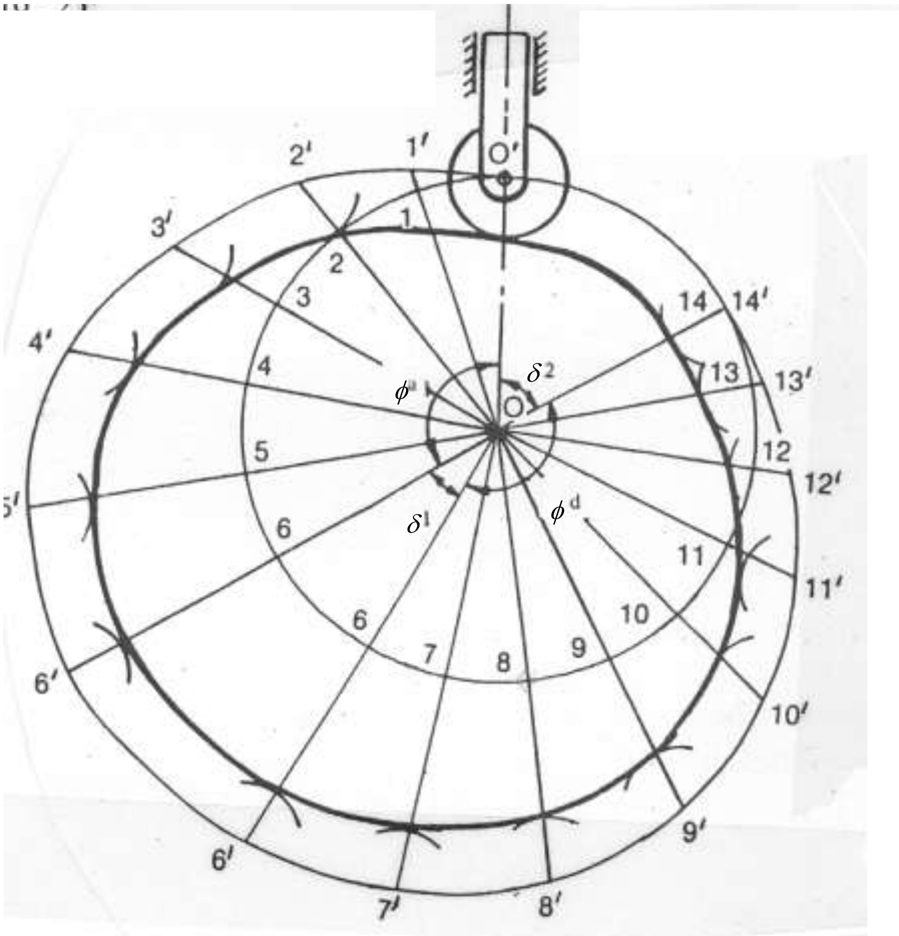

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Figure 7