Solved Practice problems

Question 1

The can in Figure 1, floats in the position shown. The can has a square cross-section.

a. What is the weight in N?
b. If the water is replaced by oil of SG= 0.8, what would be the immersed length of the can?

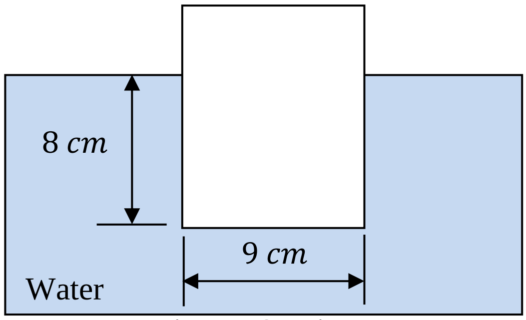


Figure 1, Question 1

Solution:

a. The weight of the can is equal to the weight of displaced fluid. Hence,
$$W = \rho_{H_2O} g \mathcal{V} = 1000 \times 9.8 \times 0.08 \times 0.09^2 = 6.35N$$
b. Since the weight is the same, then
$$W = SG_{oil} \rho_{H_2O} g \mathcal{V} = 0.8 \times 1000 \times 9.8 \times h \times 0.09^2 = 6.35N$$
$$h = \frac{6.35}{800 \times 9.8 \times 0.09^2} = 0.1\, m$$

Question 2

Given a proposed stream function,
$$\psi(x, y) = \frac{U}{L^2}(3x^2 y - y^3)$$

1. Calculate u and v.
2. Does this velocity field satisfy the conservation of mass for incompressible flow?
3. Calculate the acceleration of a fluid particle at $(x, y) = (1,0)$.
4. Is this flow rotational of irrotational.

Solution:

1. $u = \frac{\partial \psi}{\partial y} = \frac{U}{L^2}\,(3x^2 - 3y^2)$ and $v = -\frac{\partial \psi}{\partial x} = -\frac{U}{L^2}\,(6xy)$
2. If the velocity satisfies the conservation of mass for incompressible flow, then
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \frac{\partial}{\partial x}\left(\frac{U}{L^2}\,(3x^2 - 3y^2)\right) + \frac{\partial}{\partial y}\left(-\frac{U}{L^2}\,(6xy)\right)$$
$$\left(\frac{U}{L^2}\,(6x)\right) + \left(-\frac{U}{L^2}\,(6x)\right) = 0$$
Yes it satisfies the conservation of mass for incompressible flow.
3. The acceleration is given by $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla(\vec{V})$
$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$
$$a_x = 0 + \frac{U}{L^2}(3x^2 - 3y^2)\left(\frac{U}{L^2}(6x)\right) + \left(-\frac{U}{L^2}\right)(6xy)\left(\frac{U}{L^2}(-6y)\right) = 18\left(\frac{U}{L^2}\right)^2$$
$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$
$$a_y = 0 + \frac{U}{L^2}(3x^2 - y^2)\left(-\frac{U}{L^2}(6y)\right) + \left(-\frac{U}{L^2}(6xy)\right)\left(-\frac{U}{L^2}(6x)\right) = 0$$
4. For irrotational flow, $\nabla \times \vec{V} = 0$,
$$\nabla \times \vec{V} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

$$\nabla \times \vec{V} = \left(\left(-\frac{U}{L^2}(6y)\right) - \left(\frac{U}{L^2}(-6y)\right)\right)\hat{k} = \frac{6U}{L^2}\big((-y) - (-y)\big)\hat{k} = 0\hat{k}$$

Then the flow is irrotational.

Question 3

For the air flowing over a circular cylinder shown in Figure ,

1. Calculate the mass flow rate across the horizontal surfaces between sections 1 and 2.
2. If the static pressure at sections 1 and 2 is constant and equal to 100kPa calculate the total pressure at points a and b.
3. Calculate the drag force coefficient for the cylinder per span L.

$$C_D = \frac{F}{\frac{1}{2}\rho U^2 DL}$$

Where F is the force acting on the cylinder, $U = 10\,m/s,\ D = 0.15m$, $\rho_{air} = 1.2\,kg/m^3$and $L = 1\,m$.

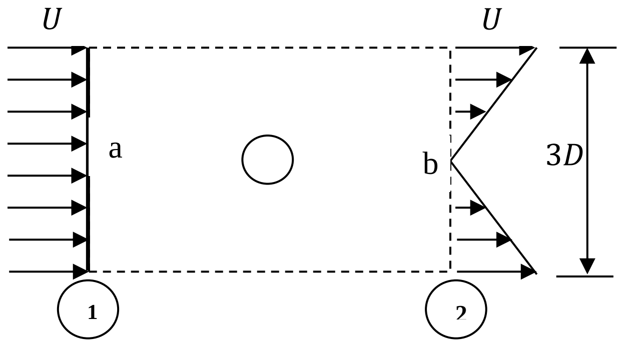


Figure 2, Question 3.

Solution:

1. $\dot{m} = \int_1 \rho\vec{V}\cdot d\vec{A} - \int_2 \rho\vec{V}\cdot d\vec{A} = \rho U(3D\times L) - 2\rho\int_0^{1.5D}\left(\frac{2U}{3D}y\right)Ldy$

$$\dot{m} = \rho U(3D\times L) - \frac{4\rho UL}{3D}\int_0^{1.5D} ydy = \rho U(3D\times L) - \frac{4\rho UL}{3D}\left(\frac{y^2}{2}\right)_0^{1.5D}$$

$$\dot{m} = \rho U(3D\times L) - \frac{4\rho UL}{3D}\left(\frac{9}{8}D^2\right) = \frac{3\rho UDL}{2} = \frac{3\times 1.2\times 10\times 0.15\times 1}{2}$$

$$\dot{m} = 2.7\ kg/s$$

2. $p_{total_a} = p_{atm} + \frac{1}{2}\rho U^2 = 10^5 + 0.5\times 1.2\times 10^2 = 100.06\,kPa$

   $p_{total_b} = p_{atm} + 0 = 10^5 = 100\,kPa$

3. The drag force, *F*, is the force due to fluid acting on the cylinder in the flow direction, hence,

$$\frac{\partial}{\partial t}\int_{CV}\rho u d\mathcal{V} + \int_{CS}\rho u\,\vec{V}\cdot d\vec{A} = -\int_{CS} pd\vec{A} - F$$

Since flow is steady and pressures are equal on all surfaces, then,

$$\int_{CS}\rho u\,\vec{V}\cdot d\vec{A} = -F$$

$$\int_1 \rho u\,\vec{V}\cdot d\vec{A} + \int_2 \rho u\,\vec{V}\cdot d\vec{A} + \int_{horizontal}\rho u\,\vec{V}\cdot d\vec{A} = -F$$

$$\int_1 \rho U\,(U\hat{\imath})\cdot(-\hat{\imath})Ldy + 2\int_0^{1.5D}\rho\left(\frac{2Uy}{3D}\right)\left(\left(\frac{2Uy}{3D}\right)\hat{\imath}\right)\cdot Ldy(\hat{\imath}) + \int_{upper}\rho U\,(U\hat{\imath} + v\hat{\jmath})\cdot Ldy(\hat{\jmath}) + \int_{lower}\rho U\,(U\hat{\imath} - v\hat{\jmath})\cdot Ldy(-\hat{\jmath}) = -F$$

$$-\rho U^2 L\int_1 dy + 2\left(\frac{4\rho U^2 L}{9D^2}\right)\int_0^{1.5D} y^2dy + \rho UL\left[\int_{upper} vdy + \int_{lower} vdy\right] = -F$$

Since $\rho L\left[\int_{upper} vdy + \int_{lower} vdy\right] = \dot{m}$ across the horizontal surfaces, then

$$-\rho U^2 L \int_1 dy + 2\left(\frac{4\rho U^2 L}{9D^2}\right)\int_2 y^2 dy + \frac{3\rho U^2 DL}{2} = -F$$
$$F = \rho U^2 L \int_{-1.5D}^{1.5D} dy - 2\left(\frac{4\rho U^2 L}{9D^2}\right)\int_2 y^2 dy - \frac{3\rho U^2 DL}{2}$$
$$F = \rho U^2 L (y)_{-1.5D}^{1.5D} - 2\left(\frac{4\rho U^2 L}{9D^2}\right)\left(\frac{y^3}{3}\right)_0^{1.5D} - \frac{3\rho U^2 DL}{2}$$
$$F = \rho U^2 L (3D) - 2\left(\frac{4\rho U^2 L}{9D^2}\right)\left(\frac{27D^3}{8}\right) - \frac{3\rho U^2 DL}{2}$$
$$F = 3\rho U^2 DL - \rho U^2 DL - \frac{3\rho U^2 DL}{2} = \frac{\rho U^2 DL}{2}$$

Hence the drag coefficient is given by,

$$C_D = \frac{F}{\frac{1}{2}\rho U^2 DL} = \frac{\frac{\rho U^2 DL}{2}}{\frac{1}{2}\rho U^2 DL} = \frac{2\rho U^2 DL}{2\rho U^2 DL} = 1$$

Question 4

Water flows steadily through the shown reducing bend in a horizontal plane with a volume flow rate of Q= 2.5 L/s discharging to the atmosphere. Assuming frictionless flow calculate:

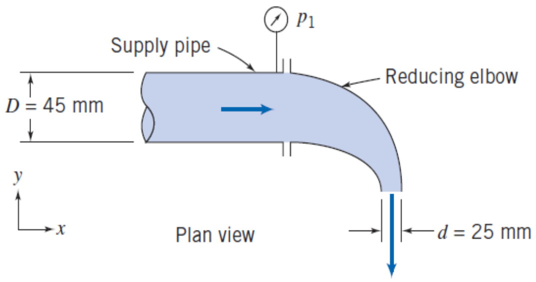


Figure 3, Question 4.

a. The velocity in the supply pipe and at the exit
b. The supply gage pressure
c. The x-component of the force exerted by the elbow on the pipe

Solution:

a. $V_{supply} = \frac{Q}{A_{supply}} = \frac{2.5\times10^{-3}}{\frac{\pi}{4}\times45^2\times10^{-6}} = 1.572\ m/s$, $V_{exit} = \left(\frac{A_{supply}}{A_{exit}}\right)V_{supply} = \left(\frac{D_{supply}}{D_{exit}}\right)^2 V_{exit} = \left(\frac{45}{25}\right)^2 \times 1.572 = 5.1\ m/s$

b. Since the flow is steady incompressible and frictionless we can apply Bernoulli's equation with z=0 since the flow is in a horizontal plane.

$$p_{supply} + \frac{1}{2}\rho V_{supply}^2 = p_{exit} + \frac{1}{2}\rho V_{exit}^2$$

Since the exit pressure is atmospheric, then $p_{supply_{gage}} = p_{supply} - p_{exit}$

$$p_{supply_{gage}} = p_{supply} - p_{exit} = \frac{\rho}{2}\left(V_{exit}^2 - V_{supply}^2\right) = \frac{1000}{2} \times (5.1^2 - 1.572^2)$$
$$p_{supply_{gage}} = 500 \times 23.54 = 11.77\ kPa$$

c. Applying the momentum equation on the bend, assuming the force to be R_x

For steady flow,

$$\int_{CS} \rho u \vec{V}\cdot d\vec{A} = \left(-\int_{CS} p d\vec{A}\right)_x + R_x$$
$$\int_{CS} \rho u \vec{V}\cdot d\vec{A} = -\int_{supply} \rho V_{supply}^2 dA + \int_{exit} \rho(0)\vec{V}_{exit}\cdot d\vec{A} = -\rho V_{supply}^2 A_{supply}$$
$$\left(-\int_{CS} p d\vec{A}\right)_x = -\int_{supply} p(-dA) = \int_{supply} p dA$$

$$R_x = -(pA)_{supply} - \rho V_{supply}^2 A_{supply} = -(p + \rho V_{supply}^2)A_{supply}$$

$$R_x = -(11.77 \times 10^3 + 10^3 \times 1.572^2) \times \frac{\pi}{4} \times (0.045^2)$$

$$R_x = -22.65\ N$$

The force by the elbow on the pipe would be in the opposite direction,

$$F_x = -R_x = 22.65\ N$$

Question 5

The drag on an airship depends on its speed V, air density ρ and viscosity μ, max diameter D and length L, i.e.

$F_d = f(V, \rho, \mu, D, L)$

a. Develop the corresponding dimensionless groups.
b. Consider a small airship traveling at 10 m/s to broadcast an event. A geometrically similar model with a scale ratio 1:10 is tested in a wind tunnel in air at the same temperature as the prototype. If dynamic similarity is to be maintained, find:
   i. Air speed in wind tunnel
   ii. The drag force of the prototype if the measured drag is 500 N
   iii. Evaluate the Mach number in the wind tunnel if the speed of sound is 340 m/s; would it have an effect on the results and why?

Solution:

a) Repeating variables ρ, V, D

Since we have six independent variables and 3 primary dimensions we will have 3 dimensionless groups,

$$\Pi_1 = \rho^a V^b D^c F_d$$

$$\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T}\right)^b (L)^c \left(\frac{ML}{T^2}\right) = M^0 L^0 T^0$$

Equating the exponents on each side

$$a + 1 = 0$$

$$-3a + b + c + 1 = 0$$

$$-b - 2 = 0$$

Solving we get $a = -1, b = -2, c = -2$, hence the first group is,

$$\Pi_1 = \frac{F_d}{\rho V^2 D^2}$$

Similarly the second group is, $\Pi_2 = \rho^d V^e D^f \mu$

$$\left(\frac{M}{L^3}\right)^d \left(\frac{L}{T}\right)^e (L)^f \left(\frac{M}{LT}\right) = M^0 L^0 T^0$$

$$d + 1 = 0$$

$$-3d + e + f - 1 = 0$$

$$-e - 1 = 0$$

Solving we get $d = -1, e = -1, c = -1$, hence the first group is,

$$\Pi_2 = \frac{\mu}{\rho V D}$$

Finaly the last group can be obtained by inspection since L is a length, then the dimensionless group is,

$$\Pi_3 = \frac{L}{D}$$

b) i) For the two flows to be similar we must have $Re_{prototype} = Re_{model}$, then

$$\frac{V_{prototype} D_{prototype}}{\nu} = \frac{V_{model} D_{model}}{\nu}$$

Then $V_{model} = \left(\frac{D_{prototype}}{D_{model}}\right) V_{prototype} = \left(\frac{10}{1}\right) 10 = 100\ m/s$

ii)The drag force is given by

$$\frac{F_{model}}{F_{prototype}} = \frac{\rho V_{model}^2 D_{model}^2}{\rho V_{prototype}^2 D_{prototype}^2} = 1$$

Then $F_{model} = F_{prototype} = 500\ N$

ii) The Mach number is given by $M_{model} = \frac{V_{model}}{a} = \frac{100}{340} = 0.294$

It will not have an effect since it is less than 0.3 so compressibility effects are not significant.

Question 6

In the shown arrangement water discharges from galvanized steel pipes (e= .15 mm) into the atmosphere, L= 10 m, D= 50 mm and H= 10m. If the tank diameter is much larger pipe diameters, calculate the flow rate of pipe A and pipe B. Note that K_{ent}= 0.5 and μ=.001 Pa.s.

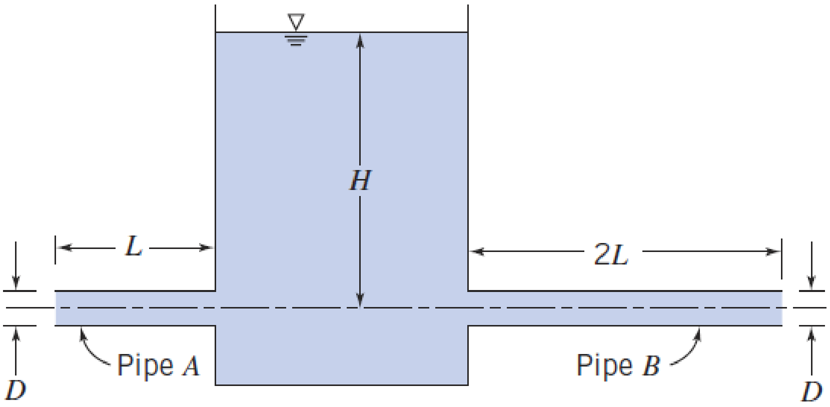


Figure 4: Question 6.

Solution:

Assumption, flow is steady velocity of water surface inside the tank is negligible.

For pipe A:

$$p_{atm} + \underbrace{\frac{1}{2}\rho V_{surface}^2}_{\approx 0} + \rho g H = p_{atm} + \frac{1}{2}\rho V_A^2 + \left(f\left(\frac{L}{D}\right)_A + K_{ent}\right)\frac{\rho V_A^2}{2}$$

$$V_A = \sqrt{\frac{2gH}{1 + f\left(\frac{L}{D}\right)_A + K_{ent}}} = \frac{14}{(1.5 + 200f)^{0.5}}$$

For $\frac{e}{D} = \frac{0.15}{50} = 0.003$ from Moody chart

$$f = 0.026 \rightarrow V_A = 5.409\ m/s \rightarrow Re = \frac{V_A D}{\nu} = 270433$$

$$Re = \frac{V_A D}{\nu} = 270433, \frac{e}{D} = 0.003 \rightarrow f = 0.027 \rightarrow V_A = 5.33\,m/s \rightarrow Re = 266485$$

$$Re = 266485, \frac{e}{D} = 0.03 \rightarrow f = 0.0265 \rightarrow V_A = 5.37\ m/s$$

Then $Q_A = V_A A_A = 5.37 \times \frac{\pi}{4} \times 0.05^2 = 0.010544\ m^3/s$

Similarly for pipe B:

$$V_B = \sqrt{\frac{2gH}{1 + f\left(\frac{L}{D}\right)_B + K_{ent}}} = \frac{14}{(1.5 + 400f)^{0.5}}$$

Since e/D is the same as well as the pressure drop then the flow rate is less than pipe A as a result the flow is probably not fully turbulent, hence f>0.026

$$f = 0.03 \rightarrow V_B = 3.81\ m/s \rightarrow\ Re = \frac{V_B D}{\nu} = 190515$$

$$Re = 190515, \frac{e}{D} = 0.003 \rightarrow f = 0.0268 \rightarrow V_A = 4\, m/s \rightarrow Re = 200000$$

$$Re = 200000, \frac{e}{D} = 0.003 \rightarrow f = 0.0266 \rightarrow V_A = 4.02\, m/s \rightarrow Re = 200900$$

Then $Q_B = V_B A_B = 4.02 \times \frac{\pi}{4} \times 0.05^2 = 0.007893\ m^3/s$

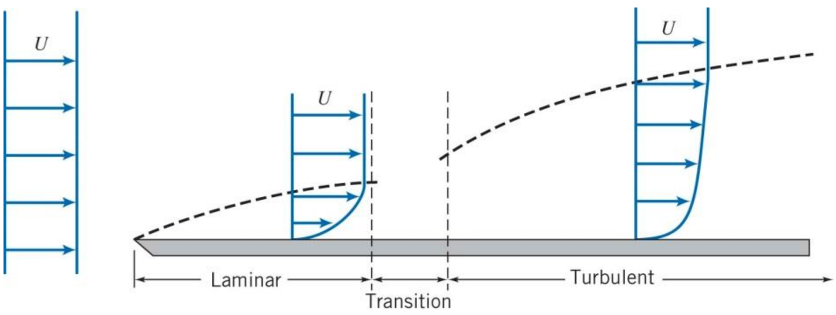
Question 7

a. Sketch the growth of the boundary layer on a flat plate.
b. Sketch the velocity profiles of laminar and turbulent boundary layers.
c. An approximate velocity profile for the laminar boundary layer on a flat plate is given by:

$$\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

i. Show what conditions at the wall and at boundary layer edge this profile satisfies.
ii. Evaluate δ^*/δ and θ/δ and hence determine the shape factor H.

Solution:



c. $y = 0,\ u = 0$

$y = \delta,\ u = U,\ \frac{du}{dy} = 0$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \frac{3}{2}\left(\frac{y}{\delta}\right) + \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right) dy$$

$$\delta^* = \left[y - \frac{3}{4}\frac{y^2}{\delta} + \frac{1}{8}\frac{y^4}{\delta^3}\right]_0^\delta = \left[\delta - \frac{3}{4}\delta + \frac{1}{8}\delta\right] = \frac{3}{4}\delta$$

$$\frac{\delta^*}{\delta} = \frac{3}{8} = 0.375$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(\frac{u}{U} - \left(\frac{u}{U}\right)^2\right) dy$$

$$\theta = \int_0^\delta \left(\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 - \left(\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right)^2\right) dy$$

$$\theta = \int_0^\delta \left[\frac{3y}{2\delta} - \frac{y^3}{2\delta^3} - \left(\frac{9y^2}{4\delta^2} - \frac{3y^4}{2\delta^4} + \frac{y^6}{4\delta^6}\right)\right] dy$$

$$\theta = \left[\frac{3y^2}{4\delta} - \frac{y^4}{8\delta^3} - \left(\frac{9y^3}{12\delta^2} - \frac{3y^4}{10\delta^4} + \frac{y^7}{28\delta^6}\right)\right]_0^\delta$$

$$\theta = \left[\frac{3}{4} - \frac{1}{8} - \frac{2520 - 1008 + 120}{3360}\right]\delta = \left[\frac{2100}{3360} - \frac{1632}{3360}\right]\delta = \left[\frac{117}{840}\right]\delta$$

$$\frac{\theta}{\delta} = \frac{39}{280} = 0.1393$$

Then the shape factor is given by,

$$H = \frac{\delta^*}{\theta} = \frac{3}{8} \times \frac{280}{39} = 2.69$$