**Math 481 2nd Semester 2017-2018**

**Mid-Exam 2 4/3/2018**

**Duration: 90 Minutes**

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| --- | --- | --- | --- | --- | --- |
| Question | 1 | 2 | 3 | 4 | total |
| Mark |  |  |  |  |  |
| Full mark | 4 | 5 | 6 | 5 | 20 |

**Question 1:**

1. (2 points) Let be defined by .
   1. Is convergent? Explain your answer.
   2. Is uniformly convergent? Explain your answer.

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**Question 2:** Prove the following:

a) (1 point) If then

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**Question 3:**

1. (1 points) State the **Cauchy Critertion for Uniform Convergence.**
2. ( 3 point) Give examples of **(Answer only 3)**
   1. Functions , with But (You can define the domain of the functions as you like).
   2. but .
   3. but
   4. but . (Example 9.8 p 316)

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**Question 4:**

1. (3 marks) Prove the Weierstrass M-Test:

Let be a sequence of functions defined on and a sequence of numbers such that for all and for all If converges, then and converges uniformly.

1. (2 Marks) Discuss where converges a) uniformly, b) absolutely, c) pointwise.
   1. .

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