King Saud University
College of Sciences
Department of Mathematics

Final Exam, S1 1439/1440
M 380 – Stochastic Processes
Time: 3 hours – Female Section

Answer the following questions:

Q1: [4+4]

(a) Suppose X and Y are jointly distributed random variables having the density function $f_{XY}(x,y) = \frac{1}{y} e^{-(x/y)-y}$ for $x, y > 0$. Find the conditional probability of X given that $Y=y$ and determine the expected value for X.

b) For the Markov process $\{X_t\}$, $t=0,1,2,\dots,n$ with states $i_0, i_1, i_2, \dots, i_{n-1}, i_n$

Prove that: $\Pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\} = p_{i_0} P_{i_0 i_1} P_{i_1 i_2} \dots P_{i_{n-1} i_n}$ where $p_{i_0} = \text{pr}\{X_0 = i_0\}$

Q2: [2+4]

a) Let X_n denote the quality of the nth item that produced in a certain factory with $X_n = 0$ meaning "good" and $X_n = 1$ meaning "defective". Suppose that $\{X_n\}$ be a Markov chain whose transition matrix is

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \left\|\begin{array}{cc} 0.98 & 0.02 \\ 0.14 & 0.86 \end{array}\right\| \end{array}$$

In the long run, what is the probability that an item produced by this system is good?

b) Consider a spare parts inventory model in which either 0, 1, or 2 repair parts are demanded in any period, with $\Pr\{\xi_n = 0\} = 0.4$, $\Pr\{\xi_n = 1\} = 0.3$, $\Pr\{\xi_n = 2\} = 0.3$ and suppose $s=0$ and $S=3$. Determine the transition probability matrix for the Markov chain $\{X_n\}$, where X_n is defined to be the quantity on hand at the end of period n.

Q3: [8]

An airline reservation system has two computers, only one of which is in operation at any given time. A computer may break down on any given day with probability p. There is a single repair facility that takes 2 days to restore a computer to normal. The facilities are such that only one computer at a time can be dealt with. Form a Markov chain by taking as states the pairs (x,y), where x is the number of machines in operating condition at the end of a day and y is 1 if a day's labor has been expended on a machine not yet repaired and 0 otherwise. Also, find the system availability.

Q4: [5+4]

(a) From purchase to purchase, a particular customer switches brands among products A, B, and C according to a Markov chain whose transition probability matrix is

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \left\| \begin{array}{ccc} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{array} \right\| \end{array}$$

In the long run, what fraction of time does this customer purchase brand A?

(b) Let $X(t)$ be a Yule process that is observed at a random time U, where U is uniformly distributed over [0,1). Show that $pr\{X(U)=k\}=p^k/(\beta k)$ for $k=1,2,\ldots$, with $p=1-e^{-\beta}$.

Q5: [5+4]

(a) Using the differential equations

$$\frac{dp_0(t)}{dt} = -\lambda p_0(t) \qquad (1)$$

$$\frac{dp_n(t)}{dt} = \lambda p_{n-1}(t) - \lambda p_n(t), \quad n=1,2,3,\ldots \qquad (2)$$

where all birth parameters are the same constant λ with initial condition $X(0)=0$,

Show that $p_n(t) = \dfrac{(\lambda t)^n e^{-\lambda t}}{n!}$, $n=0,1,2,\ldots$

(b) Let X and Y be independent Poisson distributed random variables with parameters α and β, respectively. Determine the conditional distribution of X, given that $N = X+Y = n$.