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| Math 3731st midterm examinationFirst semester, 1430HDepartment of MathematicsCollege of SciencesKing Saud University |

**Q1.**Prove or disprove the following statements:

1. A subset *U* is open in $R$ with the usual topology iff *U* is equal to a union of open intervals
2. The intersection of any collection of closed sets is closed.
3. If *A* is a subset of a topological space, then $∅$.

**Q2.** Let X be a nonempty set.

1. Show that the collection $l$= {:$∅$, or  is a countable set} is a topology on *X*; called the cocountable topology.
2. Describe closed sets in this topology.
3. Let *X =* $R$ with the cocountable topology and , describe the following:

**(i)** **(ii)** **(iii)** **(iv)** **(v)** 

1. Prove that ($R$ *,* $U$) is finer than ($R$ ,$l$).

**Q3**. **(a)** Let be a topological space with  and , show that .

 **(b)** If X= {a , b , c}, let $F\_{1}$={X , $∅$ , {a} , {a , b}} and $F\_{2}$= {X , $∅$, {a} , {b , c}}*,* find the smallest topology containing $F\_{1}$ and $F\_{2}$ and the largest topology contained in $F\_{1}$ and $F\_{2}$ .

 **(c)** Let *A* be a subset of a topological space$ (X,F)$. If $∅$ , show that *A* is dense in *X*.

 Good Luck