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|  | **King Saud University** |  |
| **College of Sciences** |  |
| **Department of Mathematics** |  |
| **Math 373** |  |
| **Second Midterm** |  |
| **Second Semester 1437-1438** |  |

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| Student’s Name:Student’s ID: |

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| **Question** | MARK |
| **1** |  |
| **2** |  |
| **3** |  |
| **4** |  |
| **Total** |  |

**Question 1:**

**Let** $(X,τ)$ **be a topological space and** $A⊆X$**.**

1. **Prove that** $τ\_{A}=\{U∩A:U\in τ\}$ **is a topology for the set** $A$**.**
2. **Is** $τ\_{A}⊆τ$**? Justify your answer.**

**Question 2:**

1. **Let** $\left(X,τ\_{X}\right),\left(Y,τ\_{Y}\right) $**be topological spaces, and let**$A⊆X$ **and** $B⊆Y$**. Prove that** $Int\left(A×B\right)=Int(A)×Int(B)$**.**
2. **Let** $f:X\rightarrow Y$ **and** $g:Y\rightarrow Z$ **be continuous functions. Prove that** $g∘f$ **is a continuous function.**
3. **Let** $f:X\rightarrow Y$ **be a bijective function. Prove that** $f$ **is a homeomorphism if and only if** $f\left(\overbar{A}\right)=\overbar{f(A)}$ **for each subset** $A⊆X$**.**

**Question 3:**

1. **Show that** $R$ **with Co-finite topology is a compact but** $R$ **with discrete topology is not.**
2. **Prove that a closed subset of a compact space is compact. Give an example to show the convers is not true.**

 **Question 4:**

1. **Define what you mean by a Hausdorff space.**
2. **Show that the property of being a Hausdorff space is a topological property.**