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|  | **King Saud University** |  |
| **College of Sciences** |  |
| **Department of Mathematics** |  |
| **Math 373** |  |
| **Final Exam** |  |
| **Second Semester 1437-1438** |  |

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| Student’s Name:Student’s ID: |

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| **Question** | MARK |
| **1** |  |
| **2** |  |
| **3** |  |
| **Total** |  |

**Question 1:**

1. **Show that the function** $d:R^{2}×R^{2}\rightarrow R$ **given by**

$$d\left(\left(x\_{1},y\_{1}\right),\left(x\_{2},y\_{2}\right)\right)=\max\_{}\{\left|x\_{1}-x\_{2}\right|,\left|y\_{1}-y\_{2}\right|\}$$

**Is a metric on** $R^{2}$**.**

1. **Prove that If** $A$ **is a compact subset of a metric space** $(X,d)$**, then** $A$ **is closed and bounded.**
2. **What do we mean by the metrizability problem? Is every topological space metrizable? (Justify your answer)**

**Question 2:**

1. **Show that if** $X $ **is a Hausdorff space, then any sequence in** $X$ **either converges to a unique point in** $X$ **or does not converge.**
2. **Is (II) true if** $X$ **isn’t Hausdorff. Justify your answer.**

**Question 3:**

1. **Prove that the compactness is a topological property.**