Definite Integral
Math 106
Lecture 2

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Let f be a function defined on $[a, b]$. The Definite Integral for f from a to b is

$$\int_a^b f(x)dx = \lim_{||p|| \to \infty} \sum_{k=1}^{n} f(w_k)x_k.$$

The numbers a and b are called the limits of the integration.

Using the above definition we can write the following:

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  $$\int_3^5 f(x^2 + x - 55)dx.$$

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If we have $b > a$, then

$$\int_a^b f(x)dx = -\int_b^a f(x)dx.$$

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If $f(x)$ exist, then

$$\int_a^a f(x)dx = 0.$$

Remarks:

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  If a function f is monotonic function (increasing or decreasing) on $[a, b]$. Then f is integrable on $[a, b]$.

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- $$\int_a^b |f(x) \pm g(x)|dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

- If the functions f, g are integrable on $[a, b]$ and $f(x) \geq g(x)$ for any $x \in [a, b]$. Then

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- Let $c \in [a, b]$. If f is integrable on $[a, c]$ and $[c, b]$. Then f is integrable on $[a, b]$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx.$$

Example: Let

$$f(x) = 4x^3 + 2, \qquad x < 0$$

$$f(x) = x - 5, \qquad x \geq 0$$

Find the following

$$\int_{-1}^{2} f(x)dx?$$

Thanks for listening.