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Question 1: (5x3=15 marks)

Prove or disprove the following:

1. If A is a subset of a topological space $(X,τ)$, then $A^{'}⊆Bd(A)$.
2. Let A and B be subsets of a topological space $(X,τ)$ with $A⊆B$, then $τ\_{A}⊆τ\_{B}.$
3. Let A and B be subsets of a topological space$(X,τ), Bd\left(A∩B\right)=Bd(A)∩Bd(B)$.
4. The co-finite topology is weaker than the usual topology on $R.$
5. No topology on $R$ has a proper subset which is both open and closed.

Question 2: (5+2+5=12 marks)

Let $=R$ , and let $τ=\{U⊂X|Either X-U is finite or 1\notin U\} $be a collection of subsets of $X$.

1. Prove that $τ$ is a topology on $X$.
2. Describe the closed subsets of $X$.
3. If $A=\left\{1,2\right\}$, then find $Int(A)$ , $Bd(A)$, and $A'$. Is $A$ dense in $X$.

Question 3: (2+5+6=13 marks)

1. Define a base for a topological space, and give an example.
2. Let $(X,τ)$ be a topological space and $A$ be nonempty subset of $X$. Prove that if $B$ is a base for the topology $τ$, then the collection $B\_{A}=\left\{B∩A\right|B\in B\}$ is a base for the subspace topology on $A.$
3. Let 𝐴 and 𝐵 be subsets of a topological space. Prove that $\overbar{A∪B}=\overbar{A}∪\overbar{B}.$

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 Good Luck ☺