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| **Name:** |  |
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| Question  | Marks |
| Q1(a) |  |
| Q1(b) |  |
| Q1(c) |  |
| Q1(d) |  |
| Q1(e) |  |
| Q1(f) |  |
| Q1(g) |  |
| Q1(h) |  |
| Q2(a) |  |
| Q2(b) |  |
| Q2(c) |  |
| Q3(a) |  |
| Q3(b) |  |
| Q3(c) |  |
| Q4(a) |  |
| Q4(b) |  |
| Q4(c) |  |
| Total out of 45 |  |

Question 1: (marks=3x8=24)

Prove or disprove the following:

1. Let (X, d) be a metric space and F ⊆ X be a finite subset. Then F is closed in X.
2. Let Y be a subspace of X, if A is closed in Y, and Y is closed in X, then A is closed in X.
3. If A ⊆ X and B ⊆ Y. Then $\overbar{A × B }$= $\overbar{A}$ × $\overbar{B}$ in X × Y.
4. The image of a Hausdorff space under a continuous map is Hausdorff.
5. Every subspace of a metric space which is closed and bounded is compact
6. Let $f:(X,d\_{1})\rightarrow (Y,d\_{2})$ be a map from a metric space $X$ to a metric space $Y$. If $d\_{1}$ is the discrete metric then $f$ is continuous
7. The intersection of two open balls in a metric space is an open ball.
8. Any two discrete topological spaces are homeomorphic

Question 2: (marks=2+3+2=7)

1. When do we say that a topological space $(X,τ)$ is $T\_{1}$- space? Give an example of a $T\_{1}$- space and another not $T\_{1}$- space.
2. A topological space $X$ is a $T\_{1}$-space if and only if every finite subset of $X$ is closed.
3. Show that any finite subset of a $T\_{1}$- space $(X,τ)$ doesn’t have a limit point.

Question 3: (marks=4+3+2=)

1. Let $\left(X,d\right)$ be a metric space, and $f:A\rightarrow X$ be injective function from a non-empty set $A$. Show that the function $\overbar{d}:A×A\rightarrow R$ given by $\overbar{d}\left(a,b\right)=d(f\left(a\right),f\left(b\right))$ is a metric for $A$.

1. Let $X=R$ have the topology $τ=\left\{U⊆X:1\in U or U=φ\right\}$. Let $s$ be the sequence in $X$, where$ s\_{n}=1, ∀n\in N$. Is it a convergent sequence? What is the limit if it exists?
2. Prove that a sequentially compactness is a topological property.

Question 4: (marks=2+3+2=7)

1. Define what do you mean by a complete metric space, and give an example of a complete metric and a non-complete metric.
2. Prove that every compact metric space is complete.
3. Describe Cauchy sequences in the Discrete Metric on$ R$ . Is this Discrete Metric complete?

Question 5:

1. Let $X$ be a metrizable space. Prove that $X$ is limit point compact space if and only if $X$ is sequentially compact.
2. If $X$ is not a metrizable space, then prove that the statement in I is not true.

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