| King Saud University | MATH 352 (Numerical Analysis 1) |
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| Department of Mathematics | First Midterm Exam |
| Second Semester 1438-1439 H | Duration: 2 Hours |

| Student Name | Student ID | Group Number |
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| Question Number | I | II | III | Total |
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**[I] Determine** whether the following is **True** or **False**. **Justify** your answer. [8 *Points*]

1. The **absolute** error in approximating $p = \sqrt{3}$ by $p^* = 1.732$ is $10^{-3}$ ( )

2. **If** $x = \frac{5}{6}$ and $y = \frac{7}{11}$, **then** using 4-digit chopping arithmetic, $x \oplus y = 1.469$ ( )

3. $q^* = 2.718$ approximates $q = e$ to **4 significant digits** ( )

4. The sequence $\alpha_n = \frac{n^2+4}{n^3}$ **satisfies** that $\alpha_n = 0 + O\left(\frac{1}{n}\right)$ ( )

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5. The function $F(h) = \frac{sinh}{h}$ **satisfies** that $F(h) = 1 + O(h^2)$ ( )

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6. Every **root** of $f(x) = x^3 - 2x^2 + x - 1$ is a **fixed-point** of $g(x) = \sqrt[3]{2x^2 - x + 1}$ ( )

**[II]** Let $f(x) = 3x - 2^x$ [6 *Points*]

(a) **Show** that $f$ has a root in $[0,1]$

(b) **Determine** the number of iterations necessary to solve $f(x) = 0$ on $[0,1]$ by the Bisection Method with accuracy $10^{-2}$

(c) Use three iterations of the Bisection Method to **approximate the root** of $f$ on $[0,1]$

(d) **Find a bound** for the relative error in (c)

**[III]** Let $g(x) = \sqrt{\frac{7}{x+2}}$ [6 *Points*]

(a) **Prove** that $g$ **has a unique** fixed-point in $[1,2]$

(b) **Use** the fixed-point iteration $p_n = g(p_{n-1})$ to determine a solution for $x^3 + 2x^2 - 7 = 0$ on $[1,2]$ with accuracy $10^{-2}$

Good Luck ☺