King Saud University	MATH 352 (Numerical Analysis 1)
Department of Mathematics	Final Exam
Second Semester 1438-1439 H	Duration: 3 Hours

Student Name	Student ID	Group Number

Question Number	I	II	III	IV	V	VI	VII	Total
Mark								

[I] Determine whether the following is **True** or **False**. **Justify** your answer. [8 *Points*]

1. If $p_0 = 1$, the sequence $p_n = p_{n-1} - \frac{p_{n-1}^3 - 18}{3p_{n-1}^2}$ converges to $\sqrt[3]{18}$ **faster** than the sequence

$p_n = \left(\frac{18}{p_{n-1}}\right)^{\frac{1}{2}}$ ( )

2. The Newton's method **converges quadratically** to the root $p = 1$ for

$f(x) = x^4 - x^3 - 3x^2 + 5x - 2$ ( )

3. The matrix $\begin{bmatrix} 0.04 & 58 \\ 5.3 & -7.1 \end{bmatrix}$ is **well-conditioned** ( )

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4. If $C = \begin{bmatrix} 10 & -4 \\ 5 & 1 \end{bmatrix}$ then $\boldsymbol{\rho(C) = 6}$ ( )

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5. The matrix $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ is **diagonally dominant** ( )

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6. The sequence $\left(\frac{3}{k^2}, \frac{\cos k}{k}, e^{-k}\right)^T$ **converges** to $(0,0,0)^T$ as $k \to \infty$ ( )

[II] Choose the correct answer. [9 *Points*]

1. The **degree of precision** for the quadrature formula $\int_{-1}^{1} f(x)dx = f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right)$ is

(a) 1 **(b)** 2 **(c)** 3 **(d)** None of the previous

2. If P_n is the Lagrange polynomial with degree 2 which interpolates $f(x) = 5^x$ using $x = -1,0,1$, then $\boldsymbol{P_n}$ **approximates** $\sqrt{\mathbf{5}}$ by

(a) 2.975 **(b)** 2.236 **(c)** 14.366 **(d)** None of the previous

3. If **Newton's Forward-Difference** formula is used to construct interpolating polynomial of degree 2 to **approximate** $\boldsymbol{f(0.25)}$ for the following data

x	0.1	0.3	0.5
$f(x)$	-0.621	0.067	0.245

Then

(a) $f(0.25) \approx -0.057$ **(b)** $f(0.25) \approx 0.344$ **(c)** $f(0.25) \approx 0.75$ **(d)** None of the previous

4. If $g(0.2) = 0.97$, $g(0.4) = 0.92$ and $g(0.6) = 0.81$, then the **backward-difference** formula to **approximate** $\boldsymbol{g'(0.4)}$ gives

(a) $g'(0.4) \approx 0.75$ **(b)** $g'(0.4) \approx 0.55$ **(c)** $g'(0.4) \approx -0.25$ **(d)** None of the previous

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5. If the **Composite Trapezoidal** rule is used to compute $\int_0^2 \frac{1}{x+4}\,dx$ with an error of at most 10^{-5}, then

(a) $n \geq 15$ **(b)** $n \geq 36$ **(c)** $n \geq 46$ **(d)** None of the previous

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6. If $M = \begin{bmatrix} 1 & 3 \\ -1 & 3 \end{bmatrix}$, then $||M||_2$ **equals**

(a) 2 **(b)** $\sqrt{2}$ **(c)** $3\sqrt{2}$ **(d)** None of the previous

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[III] If $f \in C[a, b]$ and $f(a)f(b) < 0$. **Prove** that the Bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ approximating a zero p of f with $p_n = p + O\left(\frac{1}{2^n}\right)$ [3 *Points*]

[IV] Approximate $\int_0^2 x^2 ln(x^2+1)\,dx$ with $h = 0.5$ using [5 *Points*]

(a) Composite Simpson's rule

(b) Composite Midpoint rule

[V] Let $A = \begin{bmatrix} 1.19 & 2.11 & -100 \\ 14.2 & -2.40 & 12.2 \\ 12.1 & 100 & -99.8 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -46.7 \\ 17.9 \\ 62.2 \end{bmatrix}$ [5 *Points*]

(a) Use Gaussian elimination and 3-digit **rounding** arithmetic with **scaled partial pivoting** to solve the system $A\mathbf{x} = \mathbf{b}$

(b) The exact solution of the system $A\mathbf{x} = \mathbf{b}$ is $\mathbf{x} = [1 \quad 1 \quad 0.5]^T$. **Compute** $\|\mathbf{x} - \tilde{\mathbf{x}}\|_2$ where $\tilde{\mathbf{x}}$ is the solution in (a)

[VI] Let $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ [5 *Points*]

(a) Show that A is positive definite

(b) Find an LU factorization of A where L is a lower triangular matrix and U is an upper triangular matrix.

[VII] Consider the system [7 *Points*]

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 5 \\ -x_1 + 3x_2 + x_3 &= -4 \\ 2x_1 + 2x_2 + 5x_3 &= 1 \end{aligned}$$

(a) Find the **second** iteration $\mathbf{x}^{(2)}$ of the **Gauss-Seidel** method to approximate the solution of the system using $\mathbf{x}^{(0)} = \mathbf{0}$.

(b) Write the **Jacobi** method for the given system in the matrix form $\mathbf{x}^{(k)} = T_j\mathbf{x}^{(k-1)} + \mathbf{c}_j$.

(c) Compute $\|T_j\|_\infty$.

(d) Does the Jacobi method **converge** to the solution of the given system for any choice of $\mathbf{x}^{(0)}$? **Justify** your answer.

Good Luck ☺