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| **Question Number** | **Mark** |
| **Question I** |  |
| **Question II** |  |
| **Question III** |  |
| **Question IV** |  |
| **Question V** |  |
| **Total** |  |

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| **Question I:** **A. Choose the correct answer.** **(1) The differential equation** $ \frac{d^{3}y}{dx^{3}}-\left(\frac{dy}{dx}\right)^{2}+4y=2e^{x}$ **is****(a) of order 3 and nonlinear (b) of order 3 and linear** **(c) of order 2 and linear (d) None of the previous****(2) The singular points of the differential equation**$\left(x^{2}-9\right)\frac{d^{2}y}{dx^{2}}+5x\frac{dy}{dx}+2y=0$ **are****(a)**$ x=0 $ **(b**$) $$x=-3, x=3 $**(c)** $x=0, x=-3, x=3 $ **(d) None of the previous** |
| **(3) The minimum value of the radius of convergence** $R$ **of a power series solution centered at zero of the differential equation**$\left(x^{2}+4\right)\frac{d^{2}y}{dx^{2}}+x\frac{dy}{dx}+y=0$ **is****(a)**$ R=2 $ **(b**$) $$R=0 $ **(c)** $R=\infty $ **(d) None of the previous****(4) The operator that annihilates** $(x^{2}e^{5x}+e^{-2x}cosx)$ **is****(a)** $(D-5)(D^{2}+4D+5)$ **(b)** $(D-5)^{3}+(D^{2}+4D+5) $ **(c)** $(D-5)^{3}(D^{2}+4D+5)$ **(d) None of the previous** **(5) The auxiliary equation of** $2x^{2}\frac{d^{2}y}{dx^{2}}+4x\frac{dy}{dx}+y=0$ **is** **(a)** $2m^{2}+2m+1=0$ **(b)** $2m^{2}+4m+1=0$ **(c)** $2m^{2}-4m+1=0$ **(d) None of the previous**  |
| **(6) If the auxiliary equation of a homogeneous Cauchy- Euler differential equation is**$ \left(m-1\right)^{2}=0$ **then** **(a)** $y=c\_{1}x+c\_{2}xlnx$ **(b)** $y=c\_{1}x^{-1}+c\_{2}x^{-2}$**(c)** $y=c\_{1}e^{x}+c\_{2}xe^{x}$ **(d) None of the previous** \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**(7)** $L\left\{3e^{2t}sinh4t\right\}=$**(a)**$\frac{12}{s^{2}-16}$ **(b)** $\frac{12}{(s+2)^{2}-16}$ **(c)** $\frac{12}{s^{2}-4s-12} $ **(d) None of the previous**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**(8) If the roots of a characteristic equation of a second order homogenous linear differential equation with constant coefficients are :** $1,-3, $**then the differential equation is****(a)** $\frac{d^{2}y}{dx^{2}}-2\frac{dy}{dx}+3=0$ **(b)** $\frac{d^{2}y}{dx^{2}}+2\frac{dy}{dx}-3=0$ **(c)** $\frac{d^{2}y}{dx^{2}}+2\frac{dy}{dx}-3y=0$ **(d) None of the previous** **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_****(9)** $L^{-1}\left\{\frac{1}{s-2}e^{3s}\right\}=$**(a)**$ e^{2(t-3)}U\left(t-2\right)$ **(b)** $ e^{2(t-3)}U\left(t-3\right) $**(c)** $ e^{3(t-2)}U\left(t-2\right) $ **(d) None of the previous** **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**1. **Without solving classify the differential equations below as separable, linear, exact, homogeneous and/or Bernoulli:**
2. $\frac{dy}{dx}=y\left(xy^{3}-1\right).$
3. $\left(x^{3}+y^{3}\right)dx+3xy^{2}dy=0$
4. $\left(y+x\right)dy+\left(x-y\right)dx=0$**.**

**Question II :**1. **Determine the region of the xy-plane for which the differential equation has a unique solution**$(1+y^{3})\frac{dy}{dx}=x^{2}.$
2. **Write the function** $f\left(t\right)= \left\{\begin{array}{c}2 0\leq t<3\\-2 t\geq 3 \end{array}\right. $ **in terms of unit step function, then find the Laplace transform of the given function.**

**Question III:** 1. **Solve the initial value problem**$$x^{2}\frac{dy}{dx}+x\left(x+2\right)y=e^{x} y\left(1\right)=0.$$
2. **Find the orthogonal trajectories of the family**$$c\_{1}x^{2}-y^{2}=1.$$

**Question IV:** **Solve the system of differential equations**$$\begin{matrix}(D^{2}-1)x-y=0\\(D-1)x+Dy=0\end{matrix}.$$**Question V:** **A. Find two linearly independent power series solutions about the ordinary point** $x=0$**,**$$\frac{d^{2}y}{dx^{2}}-x\frac{dy}{dx}+2y=0.$$ |
| **B. Use the Laplace transform to solve the initial value problem**$$\frac{d^{2}y}{dt^{2}}+5\frac{dy}{dt}+4y=0, y\left(0\right)=1, \frac{dy}{dt}\left(0\right)=0.$$ **Good Luck☺** |