King Saud University
Department of Mathematics
1st Semester 1432

MATH 244 (Linear Algebra)
1st Midterm Exam
Duration: 105 Minutes

Student's Name	Student's ID	Group No.	Lecturer's Name

Question No.	I	II	III	IV	Total
Mark					

[**I**] Determine whether the following is **True** or **False**. Justify your answer.

(1) If the linear system $A\mathbf{x} = \mathbf{b}$ has a unique solution, then the system $A\mathbf{x} = \mathbf{c}$ also has a unique solution. ( )

(2) If $A = [a_{ij}]_{n\times n}$ is a matrix for which $A^2 - 2A = -I_n$ and $\det(A) \neq 0$, then $A^{-1} = 2I_n - A$. ( )

(3) If $A = [a_{ij}]_{n\times n}$ is invertible then its inverse A^{-1} is unique. ( )

OVER

(4) $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, $a \neq 0$ is invertible with $A^{-1} = \frac{1}{a^2+b^2} \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$. ( )

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(5) The matrices $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -2 & 1 \end{bmatrix}$ are equivalent. ( )

[II] Choose the correct answer.

(1) If $A = [a_{ij}]_{3\times 3}$ and $B = [b_{ij}]_{3\times 3}$ is obtained from A by multiplying the first row by 4 and multiplying the third row by $\frac{3}{4}$. Then

(a) $|A| = |B|$ **(b)** $|B| = 3|A|$ **(c)** $|B| = 3^3|A|$ **(d)** None

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(2) If $A = [a_{ij}]_{n\times n}$, then

(a) $A\mathbf{adj}(A) = |A|I_n$ **(b)** $A\mathbf{adj}(A) \neq |A|I$ **(c)** $\mathbf{adj}(A) = |A|$ **(d)** None

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(3) If $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 & 0 & 1 \\ 2 & -1 & 3 & -1 \\ 4 & 0 & -2 & 0 \end{bmatrix}$, then a_{23} in AB equals

(a) -6 **(b)** -5 **(c)** 6 **(d)** None

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(4) If A and B are square matrices of the same size and AB is invertible, then

(a) A must be invertible **(b)** B must be invertible **(c)** A and B must be invertible **(d)** None

OVER

(5) If the reduced row Echelon form of $A = [a_{ij}]_{n\times n}$ is I_n, then

(a) $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions **(b)** $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $n \times 1$ matrix $\mathbf{b}$ **(c)** $|A| \neq 0$

[III]

(1) Prove the following equality where the inverses exist.

$$(C + DD^T)^{-1}D = C^{-1}D(I_n + D^TC^{-1}D)^{-1}$$

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(2) Let $A = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 & 1 \\ 0 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 3 & 5 \\ 5 & 6 \end{bmatrix}$, $D = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$. Compute the following if possible. Justify your answer.

(i) $B - C$

(ii) AB^T

(iii) $tr(D)$

(iv) A^4

(vi) $\det(A)$, $\det(A + 2D)$, $\det(AD)$.

OVER

[IV]

(1) Solve the following system for y **only** using **Cramer's Rule**.

$$\begin{aligned} x + 2y - 3z &= 9 \\ 2x - y + z &= 0 \\ 4x - y + z &= 4 \end{aligned}$$

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(2) Find the values of x, y and z for which $A = \begin{bmatrix} 2 & x - 2y + 2z & 2x + y + z \\ 3 & 5 & x + z \\ 0 & -2 & 7 \end{bmatrix}$ is symmetric.

Good Luck