King Saud University
Department of Mathematics
Second Semester 1435-1436 H

MATH 244 (Linear Algebra)
Second Midterm (Make-up) Exam
Duration: 105 Minutes

Student's Name	Student's ID	Group No.	Lecturer's Name

Question No.	I	II	III	IV	Total
Mark					

[**I**] Determine whether the following is **True** or **False**. [3 *Points*]

(1) If A and B are $n \times n$ matrices, then $\det(A^2 - B^2) = \det(A - B)\det(A + B)$. ( )

(2) If A, B and C are 3×3 matrices, such that $\det(A) = 4$, $\det(B) = -4$ and $\det(C) = 2$, then $\det(-AB^{-1}C^T) = -2$. ( )

(3) Every 2×2 matrix can be written as a linear combination of $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ ( )

(4) The angle between the vectors $(1, 2, 1)$ and $(-1, 0, 2)$ is obtuse. ( )

(5) The set $\{(2, 1, 0), (-1, 2, 3), (1, 3, 1)\}$ is a base for $\mathbb{R}^3$. ( )

(6) If the vectors v_1 and v_2 are linearly independent, then the vectors v_1, v_2 and $v_1 + v_2$ are linearly independent. ( )

OVER

[II] Choose the correct answer. [5 *Points*]

(1) If $A^3 = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 1 & 3 \\ 6 & 1 & 3 \end{bmatrix}$ then $\det(A^{-1})$ equals

(a) -2 **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{8}$ **(d)** None of the previous

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(2) The set $\{(2,1,2), (-1,0,1), u\}$ is orthogonal if

(a) $u = (2,-8,2)$ **(b)** $u = (0,-1,3)$ **(c)** $u = (1,2,3)$ **(d)** None of the previous

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(3) If $||u|| = 1$, $||u+v|| = 4$ and $d(u,v) = 6$, then $||v||$ is

(a) 2 **(b)** $\sqrt{5}$ **(c)** 5 **(d)** None of the previous

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(4) The solution space of $\begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is

(a) The origin $\{0\}$ **(b)** A line through the origin **(c)** A plane through the origin **(d)** None of the previous

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(5) Which of the following is a linear combination of $v_1 = (1,1,2)$, $(1,0,1)$, $v_3 = (2,1,3)$?

(a) $(3,1,-1)$ **(b)** $(2,4,6)$ **(c)** $(2,0,1)$ **(d)** None of the previous

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OVER

[III] [6 *Points*]

(a) Prove that $W = \{(a, b, c); a + b - c = 0\}$ is a subspace of $\mathbb{R}^3$ and find a set S, such that $W = \text{span}S$.

(b) Prove that $S = \{p_1, p_2, p_3\}$ is a basis for $\mathcal{P}_2$, where $p_1 = 2x - 3$, $p_2 = x^2 + x + 1$ and $p_3 = -x^2 + 3x + 2$. Find the coordinate vector $(p)_s$, where $p = -3x^2 + 11x - 6$.

OVER

[IV] [6 *Points*] Let $V = \mathbb{R}^2$, with the addition and scalar multiplication defined as follows:

$$(x_1, y_1) + (x_2, y_2) = ((x_1 - x_2, y_1 - y_2)$$

$$k(x, y) = (kx, ky)$$

(a) Compute $(-1, 2) + (3, 2)$ and $2(-1, 2)$;

(b) Find the object $0 \in V$, such that $u + 0 = 0 + u = u$, for all $u \in V$;

(c) If $u \in V$, find the object $-u \in V$, such that $u + (-u) = (-u) + u = 0$;

(d) Is V a vector space? Justify your answer.

Good Luck