King Saud University | Final Exam | Summer Semester
College of Science | Math244 | 1439/1440H
Department of Mathematics | | Duration: 3 hr.

Calculators are not allowed

Question 1: [3+2+2 Marks]

(a) Let
$$\begin{aligned} x_1+x_2-x_3&=1\\ 2x_1+3x_2+\alpha x_3&=3\\ x_1+\alpha x_2+3x_3&=2 \end{aligned}$$
be a given system of linear equations.

For what values of α does the system have

(i) a unique solution (ii) infinitely many solutions (iii) no solution?

(b) The matrix A satisfies $A^3+4A^2-2A+2I=\underline{0}$. Show that A is invertible.

(c) Find $\left|3(adjA)^{-1}+A\right|$ where A is a matrix of size 4×4 such that $|A|=3$.

Question 2: [3+3+3 Marks]

(a) Determine whether the following vectors span $\mathbb{R}^3$.

$$v_1=(1,4,-1),v_2=(5,-2,9),v_3=(2,-3,5),v_4=(3,1,4)$$

(b) Given the inner product space $(\mathbb{R}^2,<,>)$ where

$<(x_1,y_1),(x_2,y_2)>=2x_1x_2+y_1y_2$, and $u=(1,2),v=(1,3)$.

Verify that the Cauchy-Schwarz inequality holds.

(c) Given the inner product space $(\mathbb{R}^2,<,>)$ where

$$<(x_1,y_1),(x_2,y_2)>=\alpha x_1x_2+\beta y_1y_2$$

Find the values of α and β so that $B=\left\{v_1=\left(-1,\sqrt{3}\right),v_2=\left(1,\sqrt{3}\right)\right\}$

is an orthonormal basis for $\mathbb{R}^2$.

Question 3: [4+4+6 Marks]

(a) (i) Show that $B=\{v_1=(1,0,1),v_2=(1,1,1),v_3=(1,1,0)\}$ is a basis for $\mathbb{R}^3$.

(ii) Let $T:\mathbb{R}^3\to\mathbb{R}^2$ be the linear transformation such that

$$T(v_1)=(1,-1),T(v_2)=(1,2),T(v_3)=(1,1)$$

Find a formula for $T(x_1,x_2,x_3)$.

(b) Let $T:\mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation such that

$$[T]_B^C = \begin{bmatrix} -1 & -1 \\ 0 & 2 \\ 1 & 2 \end{bmatrix}$$

is the matrix for T with respect to the ordered bases

$B = \{v_1 = (0,1), v_2 = (1,2)\}$ and $C = \{w_1 = (1,-1,0), w_2 = (0,1,0), w_3 = (0,1,1)\}$.

Find a formula for $T(x,y)$.

(c) Let $T:\mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation given by the formula

$$T(x_1,x_2,x_3,x_4) = (x_1 - x_3 + 2x_4, -2x_1 + x_2 + 2x_3, x_2 + 4x_4)$$

(i) Find a basis for the kernel of T. (ii) Find a basis for the range of T.

Question 4: [7+3 Marks]

(a) Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 1 & -2 & 1 \end{bmatrix}$.

(i) Find the eigenvalues of A. (ii) Find bases for the eigenspaces of A.

(iii) Determine whether A is diagonizable. If so, find a matrix P that diagonalizes A, and determine $P^{-1}AP$.

(iv) Compute A^{10} .

(b) Let $A = \begin{bmatrix} a+b & a \\ -b & a-b \end{bmatrix}$. Find the values of a and b so that $v = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ is an eigenvector of A corresponding to the eigenvalue -1 .