

Calculators are not allowed

Question 1 : [8 pts]

1. Let A, B be matrices of size $(3, 3)$ such that A is not invertible and $|B| = 2$. Find $|A \text{adj}(A) + 2B^{-1}|$.

2. Compute the following determinant $A = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix}$.

3. Compute the inverse matrix of the matrix A , where $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$.

Question 2 : [8pts]

1. Given the linear system $\begin{cases} x - y + 3z + 2t = a \\ -x + 8z + 3t = a \\ -2x + y + 5z + t = b \\ 3x - 2y - 2z + t = c \end{cases}$

Find the conditions on a, b, c such that the system is consistent.

2. Given the linear system: $\begin{cases} ax + by - 3z = -3 \\ -2x - by + cz = -1 \\ ax + 3y - cz = -3 \end{cases}$

(i) Find the values of a, b , and c so that the system has the solution $x = 1, y = -1$, and $z = 2$.

(ii) Solve the system for the values of a, b, c found in (i).

Question 3 : [9pts]

1. Let $E = \{(x, y, z) \in \mathbb{R}^3; ax + y + 2z = b^2 - 4\}$.

Find $a, b \in \mathbb{R}$ such that E is a sub-space of \mathbb{R}^3 .

2. Let F be the subspace of \mathbb{R}^3 generated by the vectors $v_1 = (1, -1, 2)$, $v_2 = (0, 1, -1)$, $v_3 = (1, 0, 1)$, and $v_4 = (1, 1, 0)$.

Is the vector $v = (1, 1, 1)$ in F ? (Justify your answer.)

3. Let $W = \{(x, y, z, t) \in \mathbb{R}^4; x - 2z = 0, y + z = 0\}$.

(a) Find a basis for W .

(b) Which of the following vectors belong to W .

$u = (0, 1, -1, 1)$, $v = (2, 0, -1, 5)$, $w = (-2, 1, -1, -7)$. (Justify your answer.)