

SECOND MID TERM EXAMINATION, DECEMBER 13, 2017
DEPARTMENT OF MATHEMATICS
KING SAUD UNIVERSITY
MATH: 240 FULL MARK: 25 TIME: 1 HOUR 30 MINUTES

1. Determine the dimension of and basis for the solution space of the system

$$\begin{aligned}2a + 2b - c + x &= 0 \\ -a - b + 2c - 3d + x &= 0 \\ a + b - 2c - x &= 0 \\ c + d + x &= 0\end{aligned}$$

2. Find the rank and nullity of the matrix

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

3. Find a basis for the subspace of \mathfrak{R}^4 spanned by the vectors $\mathbf{v}_1 = (-1, 1, -2, 0)$, $\mathbf{v}_2 = (3, 3, 6, 0)$ and $\mathbf{v}_3 = (9, 0, 0, 3)$.

4. Let $T: \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$ be the linear operator defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_1 + x_2 \end{bmatrix}$$

and let $\mathcal{B} = \{\mathbf{u}_1, \mathbf{u}_2\}$ be the basis for which

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

and

$$\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- (a) Find $[T]_{\mathcal{B}}$.

- (b) Verify that the formula $[T]_{\mathcal{B}}[\mathbf{x}]_{\mathcal{B}} = [\mathbf{T}(\mathbf{x})]_{\mathcal{B}}$ holds for every vector \mathbf{x} in \mathfrak{R}^2 .