M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[1]

Solution of the First Mid-Term Exam

First semester 1436-1437 H

Q.1 Let $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 0 & 1 \\ 3 & 2 \\ 1 & 3 \end{pmatrix}$.

Compute (if possible) : **AB** and **BC**

Solution :

$$\mathbf{AB} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 2 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 3 & 2 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+6+0 & 1+4+3 \\ 3+12+0 & 3+8+2 \\ 2+0+0 & 2+0+1 \end{pmatrix} = \begin{pmatrix} 7 & 8 \\ 15 & 13 \\ 2 & 3 \end{pmatrix}$$

BC is impossible , because the number of columns of **B** does not equal the number of rows of **C**.

Q.2 Compute The determinant $\begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}$

Solution (1) : Using Sarrus Method

$$\begin{matrix} 2 & 1 & 3 & 2 & 1 \\ 1 & 2 & 1 & 1 & 2 \\ 0 & 1 & 2 & 0 & 1 \end{matrix}$$

$$\begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = (8+0+3) - (0+2+2) = 11 - 4 = 7$$

Solution (2) : By the definition (using third row)

$$\begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 0 \times \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} - 1 \times \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} + 2 \times \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 0 - (2-3) + 2(4-1) = 1 + 6 = 7$$

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Q.3 Solve by Gauss elimination : $\begin{cases} x - 2y + z = -1 \\ x + y - 2z = -1 \\ 4x + y + z = 2 \end{cases}$

Solution : The augmented matrix is

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & -1 \\ 4 & 1 & 1 & 2 \end{array}\right) \xrightarrow{-R_1+R_2} \left(\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 0 \\ 4 & 1 & 1 & 2 \end{array}\right)$$

$$\xrightarrow{-4R_1+R_3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 0 \\ 0 & 9 & -3 & 6 \end{array}\right) \xrightarrow{-3R_2+R_3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 3 & -3 & 0 \\ 0 & 0 & 6 & 6 \end{array}\right)$$

$6z = 6 \implies z = 1$

$3y - 3z = 0 \implies 3y - 3 = 0 \implies y = 1$

$x - 2y + z = -1 \implies x - 2 + 1 = -1 \implies x = 0$

The solution is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

Q.4 Find all the elements of the conic section $9y^2 + 4x^2 + 18y - 8x = 23$ and sketch it.

Solution :

$9y^2 + 4x^2 + 18y - 8x = 23$

$4x^2 - 8x + 9y^2 + 18y = 23$

$4(x^2 - 2x) + 9(y^2 + 2y) = 23$

By completing the square

$4(x^2 - 2x + 1) + 9(y^2 + 2y + 1) = 23 + 4 + 9$

$4(x - 1)^2 + 9(y + 1)^2 = 36$

$\frac{4(x-1)^2}{36} + \frac{9(y+1)^2}{36} = 1$

$\frac{(x-1)^2}{9} + \frac{(y+1)^2}{4} = 1$

The conic section is an ellipse .

The center is $P(1, -1)$.

$a^2 = 9 \implies a = 3$

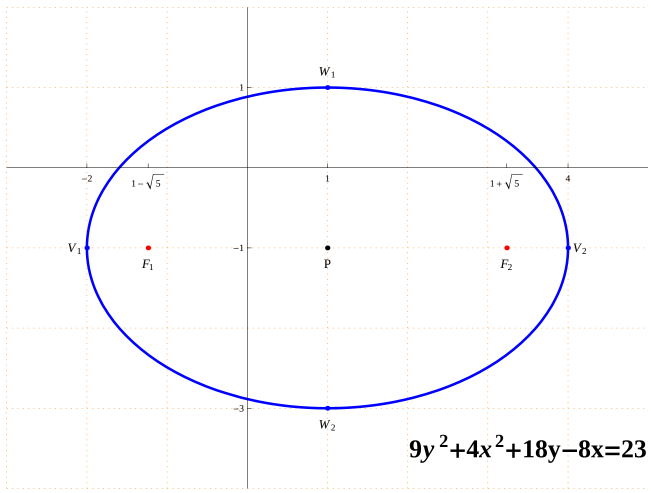
$b^2 = 4 \implies b = 2$

$c^2 = a^2 - b^2 = 9 - 4 = 5 \implies c = \sqrt{5}$

The vertices are $V_1(-2,-1)$ and $V_2(4,-1)$

The foci are $F_1\left(1-\sqrt{5},-1\right)$ and $F_2\left(1+\sqrt{5},-1\right)$

The end-points of the minor axis are $W_1(1,1)$ and $W_2(1,-3)$



Q.5 Find the standard equation of the hyperbola with foci $F_1(8,0)$ and $F_2(-8,0)$ and with vertices $V_1(5,0)$ and $V_2(-5,0)$, and then sketch it.

Solution :

Note that the two foci lie on the x-axis , hence the equation of the hyperbola has the form $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$.

The center of the hyperbola is the mid-point of the two foci (or the two vertices).

The center is $P\left(\dfrac{-8+8}{2},\dfrac{0+0}{2}\right)=(0,0)$, hence $h=0$ and $k=0$.

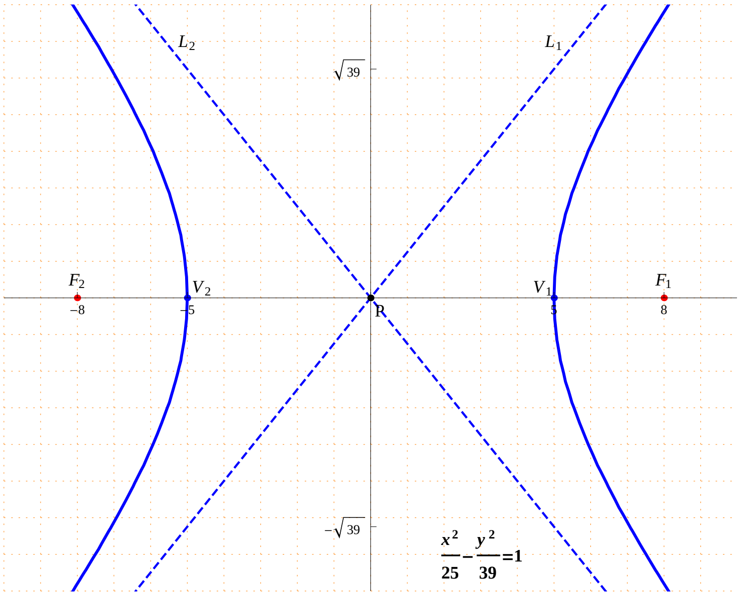
a is the distance between the center P and V_1 (or V_2) , hence $a=5$.

c is the distance between the center P and F_1 (or F_2) , hence $c=8$

$c^2=a^2+b^2 \implies 8^2=5^2+b^2 \implies b^2=64-25=39 \implies b=\sqrt{39}$.

The equation of the hyperbola is $\dfrac{x^2}{25}-\dfrac{y^2}{39}=1$.

The equations of the asymptotes are $L_1:\ y=\dfrac{\sqrt{39}}{5}x$ and $L_2:\ y=-\dfrac{\sqrt{39}}{5}x$



L_2
L_1
$\sqrt{39}$
F_2
V_2
V_1
F_1
−8
−5
P
5
8
$-\sqrt{39}$
$\frac{x^2}{25}-\frac{y^2}{39}=1$

M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[2]

Solution of the Second Mid-Term Exam

First semester 1436-1437 H

Q.1 Compute the integrals :

(a) $\int 2x(x^2+7)^{14}\ dx$

(b) $\int \frac{dx}{x^2+4x+5}$

(c) $\int \frac{x+1}{(x-3)^2(x-1)}\ dx$

(d) $\int \ln|x|\ dx$

(e) $\int_0^1 xe^x\ dx$

Solution :

(a) $\int 2x(x^2+7)^{14}\ dx = \frac{(x^2+7)^{15}}{15} + c$

Using the formula $\int [f(x)]^n f'(x)\ dx = \frac{[f(x)]^{n+1}}{n+1} + c$, where $n \neq -1$

(b) $\int \frac{dx}{x^2+4x+5} = \int \frac{1}{(x^2+4x+4)+1}\ dx = \int \frac{1}{(x+2)^2+(1)^2}\ dx$

$= \tan^{-1}(x+2) + c$

Using the formula $\int \frac{f'(x)}{a^2+[f(x)]^2}\ dx = \frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right) + c$, where $a > 0$

(c) $\int \frac{x+1}{(x-3)^2(x-1)}\ dx$

Using the method of partial fractions

$\frac{x+1}{(x-3)^2(x-1)} = \frac{A_1}{x-1} + \frac{A_2}{x-3} + \frac{A_3}{(x-3)^2}$

$x+1 = A_1(x-3)^2 + A_2(x-1)(x-3) + A_3(x-1)$

$x+1 = A_1(x^2-6x+9) + A_2(x^2-4x+3) + A_3(x-1)$

$x+1 = A_1x^2 - 6A_1x + 9A_1 + A_2x^2 - 4A_2x + 3A_2 + A_3x - A_3$

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By comparing the coefficients of both sides :

$$\begin{array}{lcl} A_1 + A_2 = 0 & \longrightarrow & (1) \\ -6A_1 - 4A_2 + A_3 = 1 & \longrightarrow & (2) \\ 9A_1 + 3A_2 - A_3 = 1 & \longrightarrow & (3) \end{array}$$

Adding the three equations : $4A_1 = 2 \implies A_1 = \frac{1}{2}$

From Equation (1) : $\frac{1}{2} + A_2 = 0 \implies A_2 = -\frac{1}{2}$

From equation (2) : $-6\left(\frac{1}{2}\right) - 4\left(-\frac{1}{2}\right) + A_3 = 1$

$\implies -3 + 2 + A_3 = 1 \implies A_3 = 2$

$$\int \frac{x+1}{(x-3)^2(x-1)}\ dx = \int \left(\frac{\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}}{x-3} + \frac{2}{(x-3)^2}\right)\ dx$$

$$= \frac{1}{2}\int \frac{1}{x-1}\ dx - \frac{1}{2}\int \frac{1}{x-3}\ dx + 2\int (x-3)^{-2}\ dx$$

$$= \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x-3| + 2\frac{(x-3)^{-1}}{-1} + c$$

(d) $\int \ln|x|\ dx$

Using integration by parts :

$$\begin{array}{ll} u = \ln|x| & dv = dx \\ du = \frac{1}{x}\ dx & v = x \end{array}$$

$$\int \ln|x|\ dx = x\ln|x| - \int x\ \frac{1}{x}\ dx = x\ln|x| - \int 1\ dx = x\ln|x| - x + c$$

(e) $\int_0^1 xe^x\ dx$

Using integration by parts :

$$\begin{array}{ll} u = x & dv = e^x dx \\ du = dx & v = e^x \end{array}$$

$$\int_0^1 xe^x\ dx = [xe^x]_0^1 - \int_0^1 e^x\ dx = [xe^x]_0^1 - [e^x]_0^1$$

$$= \left(1\times e^1 - 0\times e^0\right) - \left(e^1 - e^0\right) = (e-0) - (e-1) = e - e + 1 = 1$$

Q.2 Find the area of the region bounded by the curves :

$y = x^2$ and $y = 4$

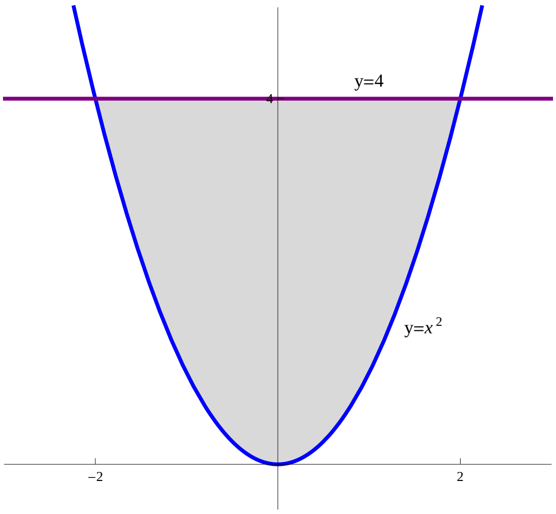
Solution :

$y = 4$ is a straight line parallel to the x-axis and passes through $(0, 4)$

$y = x^2$ is a parabola opens upwards with vertex $(0, 0)$

Points of intersection of $y = x^2$ and $y = 4$:

$x^2 = 4 \implies x = \pm 2$



$$\text{Area} = \int_{-2}^{2} (4 - x^2)\, dx = \left[4x - \frac{x^3}{3} \right]_{-2}^{2}$$

$$= \left(4 \times 2 - \frac{2^3}{3} \right) - \left(4 \times -2 - \frac{(-2)^3}{3} \right) = \left(8 - \frac{8}{3} \right) - \left(-8 + \frac{8}{3} \right)$$

$$= 16 - \frac{16}{3} = \frac{32}{3}$$

Q.3 Find the volume of the solid of revolution generated by rotation about the y-axis of the region **R** limited by the following curves :

$x = 0$, $x = 1$, $y = 1$ and $y = x^2 + 3$

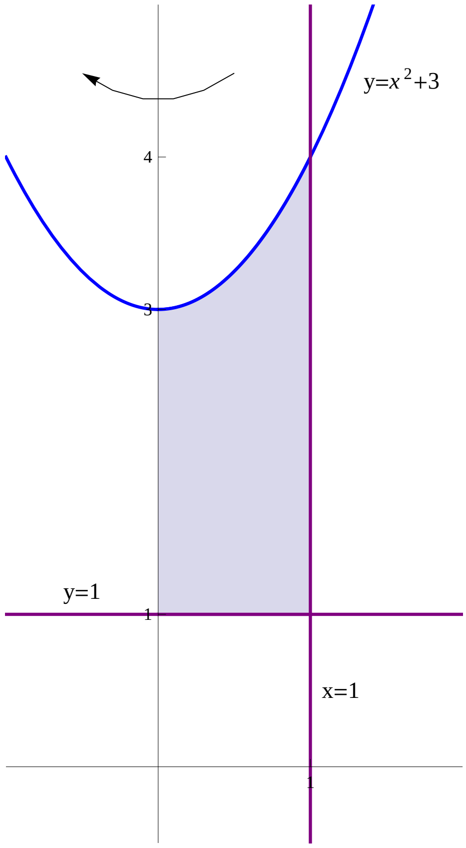
Solution :

$x = 0$ is the y-axis

$x = 1$ is a straight line parallel to the y-axis and passes through $(1, 0)$

$y = 1$ is a straight line parallel to the x-axis and passes through $(0, 1)$

$y = x^2 + 3$ is a parabola opens upwards with vertex $(0, 3)$



Using Cylindrical shells method :

$$\text{Volume} = 2\pi \int_0^1 x\left[(x^2+3)-1\right]\ dx = 2\pi \int_0^1 x(x^2+2)\ dx$$

$$= 2\pi \int_0^1 (x^3+2x)\ dx = 2\pi \left[\frac{x^4}{4} + x^2\right]_0^1$$

$$= 2\pi \left[\left(\frac{1}{4}+1\right) - (0+0)\right] = 2\pi \left(\frac{5}{4}\right) = \frac{10\pi}{4}$$

M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[3]

Solution of the Final Exam

First semester 1436-1437 H

Q.1 **(a)** Compute (if possible) **AB** for $\mathbf{A} = \begin{pmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 0 & 2 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

(b) Compute the determinant $\begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 1 \end{vmatrix}$.

(c) Solve by Cramer's rule : $\begin{array}{rcrcl} x & - & 2y & = & 0 \\ 3x & - & 5y & = & 1 \end{array}$

Solution :

(a) $\mathbf{AB} = \begin{pmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 0+0+0 & 0+2+0 & 0+0+0 \\ 3+0+3 & 0+0+0 & 3+0+3 \\ 0+0+0 & 0+2+0 & 0+0+0 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 0 \\ 6 & 0 & 6 \\ 0 & 2 & 0 \end{pmatrix}$$

(b) **Solution (1):** Using Sarrus Method

$$\begin{array}{ccccc} 1 & 0 & 2 & 1 & 0 \\ 0 & 3 & 0 & 0 & 3 \\ 4 & 0 & 1 & 4 & 0 \end{array}$$

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 1 \end{vmatrix} = (3+0+0) - (24+0+0) = 3 - 24 = -21$$

Solution (2): $\begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 1 \end{vmatrix} \xrightarrow{-4R_1+R_3} \begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & -7 \end{vmatrix} = 1 \times 3 \times -7 = -21$

Solution (3) : Using the definition of the determinant

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 1 \end{vmatrix} = 1 \times \begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} - 0 \times \begin{vmatrix} 0 & 0 \\ 4 & 1 \end{vmatrix} + 2 \times \begin{vmatrix} 0 & 3 \\ 4 & 0 \end{vmatrix}$$

$$= 1 \times (3-0) - 0 + 2 \times (0-12) = 3 - 24 = -21$$

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(c) Using Cramer's rule :

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix} , \mathbf{A}_x = \begin{pmatrix} 0 & -2 \\ 1 & -5 \end{pmatrix} , \mathbf{A}_y = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$

$$|\mathbf{A}| = \begin{vmatrix} 1 & -2 \\ 3 & -5 \end{vmatrix} = (1 \times -5) - (-2 \times 3) = -5 - (-6) = -5 + 6 = 1$$

$$|\mathbf{A}_x| = \begin{vmatrix} 0 & -2 \\ 1 & -5 \end{vmatrix} = (0 \times -5) - (-2 \times 1) = 0 - (-2) = 2$$

$$|\mathbf{A}_y| = \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} = (1 \times 1) - (0 \times 3) = 1 - 0 = 1$$

$$x = \frac{|\mathbf{A}_x|}{|\mathbf{A}|} = \frac{2}{1} = 2$$

$$y = \frac{|\mathbf{A}_y|}{|\mathbf{A}|} = \frac{1}{1} = 1$$

The solution of the linear system is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Q.2 **(a)** Find the standard equation of the ellipse with foci $(-2, 3)$ and $(4, 3)$ and the length of its major axis is 10 , and then sketch it.

(b) Find The elements of the conic section $y^2 - 4x - 2y + 13 = 0$

Solution :

(a) The standard equation of the ellipse is $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

The center $P = (h, k)$ is located between the two foci.

The center is $P = (h, k) = \left(\dfrac{-2+4}{2}, \dfrac{3+3}{2} \right) = (1, 3)$

The major axis (where the two foci are located) is parallel to the x-axis , hence $a > b$.

The length of the major axis equals 10 means $2a = 10 \implies a = 5$.

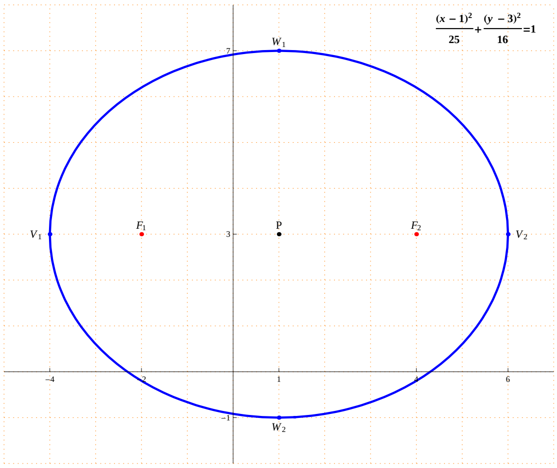
c is the distance between the two foci , hence $c = 3$

$c^2 = a^2 - b^2 \implies 9 = 25 - b^2 \implies b^2 = 25 - 9 = 16 \implies b = 4$

The standard equation of the ellipse is $\dfrac{(x-1)^2}{25} + \dfrac{(y-3)^2}{16} = 1$

The vertices are $V_1 = (-4, 3)$ and $V_2 = (6, 3)$

The end-points of the minor axis are $W_1 = (1, 7)$ and $W_2 = (1, -1)$



(b) $y^2 - 4x - 2y + 13 = 0$

$y^2 - 2y = 4x - 13$

By completing the square

$y^2 - 2y + 1 = 4x - 13 + 1$

$(y-1)^2 = 4x - 12$

$(y-1)^2 = 4(x-3)$

The conic section is a parabola opens to the right.

The vertex is $V = (3, 1)$.

$4a = 4 \implies a = 1.$

The focus is $F = (4, 1)$.

The equation of the directrix is : $x = 2$

Q.3 **(a)** Compute the integrals :

(i) $\displaystyle\int \frac{1}{(x-2)(x-1)}\ dx$ (ii) $\displaystyle\int x \sin x\ dx$ (iii) $\displaystyle\int (2x+1)(x^2+x+1)^{25}\ dx$

(b) Find the area of the surface delimited by the curves :

$y = x^2$ and $y = x$.

(c) The region R between the curves $y = 0$, $x = 1$, and $y = x^2$ is rotated about the x-axis to form a solid of revolution S . Find the volume of S.

Solution :

(a) (i) $\displaystyle\int \frac{1}{(x-2)(x-1)}\ dx$

Using the method of partial fractions

$$\frac{1}{(x-2)(x-1)} = \frac{A_1}{x-1} + \frac{A_2}{x-2}$$

$1 = A_1(x-2) + A_2(x-1)$

Put $x = 1$ then $1 = -A_1 \implies A_1 = -1$

Put $x = 2$ then $1 = A_2$

$$\int \frac{1}{(x-2)(x-1)}\ dx = \int \left(\frac{-1}{x-1} + \frac{1}{x-2}\right)\ dx$$

$$= -\int \frac{1}{x-1}\ dx + \int \frac{1}{x-2}\ dx = -\ln|x-1| + \ln|x-2| + c$$

(ii) $\int x \sin x\ dx$

Using integration by parts

$u = x \qquad dv = \sin x\ dx$
$du = dx \qquad v = -\cos x$

$$\int x \sin x\ dx = x(-\cos x) - \int -\cos x\ dx$$

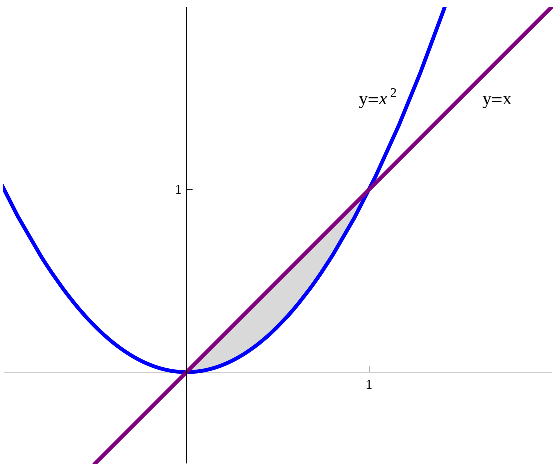
$$= -x\cos x + \int \cos x dx = -x\cos x + \sin x + c$$

(iii) $\int (2x+1)(x^2+x+1)^{25}\ dx = \frac{(x^2+x+1)^{26}}{26} + c$

Using the formula $\int [f(x)]^n\, f'(x)\ dx = \frac{[f(x)]^{n+1}}{n+1} + c$ where $n \neq -1$.

(b) $y = x$ is a straight line passes through the origin with slope equals 1.

$y = x^2$ is a parabola with vertex $(0,0)$ and opens upwards.



Points of intersection of $y = x$ and $y = x^2$:

$x^2 = x \implies x^2 - x = 0 \implies x(x-1) = 0 \implies x = 0\ ,\ x = 1$

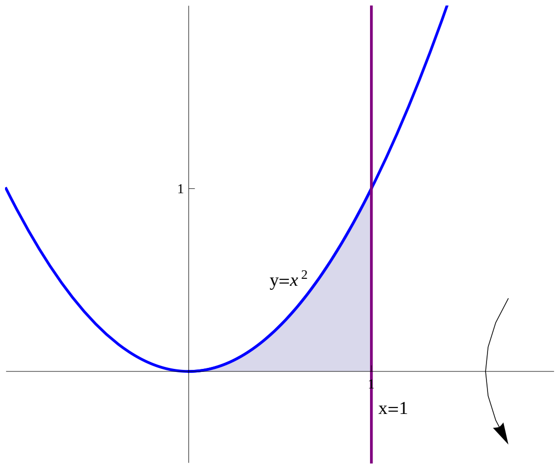
$$\text{Area} = \int_0^1 \left(x - x^2\right)\ dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{1^2}{2} - \frac{1^3}{3}\right) - (0-0) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

(c) $y = 0$ is the x-axis.

$x = 1$ is a straight line parallel to the y-axis and passes through $(1, 0)$.

$y = x^2$ is a parabola opens upwards with vertex $(0, 0)$.



Using Disk Method :

$$\text{Volume} = \pi \int_0^1 \left(x^2\right)^2\ dx = \pi \int_0^1 x^4\ dx = \pi \left[\frac{x^5}{5}\right]_0^1$$

$$= \pi \left(\frac{1^5}{5} - \frac{0^5}{5}\right) = \pi \frac{1}{5} = \frac{\pi}{5}$$

Q.4 **(a)** Find f_x and f_y for the function $f(x, y) = x^2y^3 + \sin(x+y)$

(b) Solve the differential equation $\dfrac{dy}{dx} = 2xy^2\ ,\ y(1) = 1$

Solution :

(a) $f_x = (2x)y^3 + \cos(x+y)(1+0) = 2xy^3 + \cos(x+y)$

$f_y = (3y^2)x^2 + \cos(x+y)(0+1) = 3x^2y^2 + \cos(x+y)$

(b) $\dfrac{dy}{dx} = 2xy^2$, $y(1) = 1$

$\dfrac{1}{y^2}\ dy = 2x\ dx$

$y^{-2}\ dy = 2x\ dx$

It is a separable differential equation.

$\displaystyle\int y^{-2}\ dy = \int 2x\ dx$

$\dfrac{y^{-1}}{-1} = x^2 + c$

$\dfrac{-1}{y} = 2x + c$

$y = \dfrac{-1}{x^2 + c}$

The general solution of the differential equation is $y = \dfrac{-1}{x^2 + c}$

Using the initial condition $y(1) = 1$:

$1 = \dfrac{-1}{1^2 + c} \implies 1 + c = -1 \implies c = -2$

The particular solution of the differential equation is $y = \dfrac{-1}{x^2 - 2}$

M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[4]

Solution of the First Mid-Term Exam

Second semester 1436-1437 H

Q.1 Let $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 0 & 1 \\ 3 & 6 \\ 1 & 3 \end{pmatrix}$.

Compute (if possible) : **AB** and **BC**

Solution :

$$\mathbf{AB} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 2 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+0 & 1+4+3 \\ 3+0+0 & 3+(-4)+2 \\ 2+0+0 & 2+0+1 \end{pmatrix} = \begin{pmatrix} 1 & 8 \\ 3 & 1 \\ 2 & 3 \end{pmatrix}$$

BC is impossible , because the number of columns of **B** does not equal the number of rows of **C**.

Q.2 Compute The determinant $\begin{vmatrix} 1 & 2 & -3 & 4 \\ 4 & 3 & 2 & 1 \\ -1 & -2 & 3 & -4 \\ 2 & 4 & 6 & 8 \end{vmatrix}$

Solution :

$$\begin{vmatrix} 1 & 2 & -3 & 4 \\ 4 & 3 & 2 & 1 \\ -1 & -2 & 3 & -4 \\ 2 & 4 & 6 & 8 \end{vmatrix} = 0$$

Because $R_1 = -R_3$

Q.3 Solve by Cramer's rule : $\begin{cases} x - 2y = -1 \\ 3x + y = 11 \end{cases}$

Solution :

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix} , \mathbf{A}_x = \begin{pmatrix} -1 & -2 \\ 11 & 1 \end{pmatrix} , \mathbf{A}_y = \begin{pmatrix} 1 & -1 \\ 3 & 11 \end{pmatrix}$$

$$|\mathbf{A}| = \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = (1 \times 1) - (-2 \times 3) = 1 - (-6) = 1 + 6 = 7$$

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$|\mathbf{A}_x| = \begin{vmatrix} -1 & -2 \\ 11 & 1 \end{vmatrix} = (-1 \times 1) - (-2 \times 11) = -1 - (-22) = -1 + 22 = 21$

$|\mathbf{A}_y| = \begin{vmatrix} 1 & -1 \\ 3 & 11 \end{vmatrix} = (1 \times 11) - (-1 \times 3) = 11 - (-3) = 11 + 3 = 14$

$x = \dfrac{|\mathbf{A}_x|}{|\mathbf{A}|} = \dfrac{21}{7} = 3$

$y = \dfrac{|\mathbf{A}_y|}{|\mathbf{A}|} = \dfrac{14}{7} = 2$

The solution is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Q.4 Find all the elements of the conic section $y^2 + 4x^2 + 2y - 8x + 1 = 0$ and sketch it.

Solution :

$y^2 + 4x^2 + 2y - 8x + 1 = 0$

$4x^2 - 8x + y^2 + 2y = -1$

$4(x^2 - 2x) + (y^2 + 2y) = -1$

By completing the square

$4(x^2 - 2x + 1) + (y^2 + 2y + 1) = -1 + 4 + 1$

$4(x - 1)^2 + (y + 1)^2 = 4$

$\dfrac{4(x-1)^2}{4} + \dfrac{(y+1)^2}{4} = 1$

$\dfrac{(x-1)^2}{1} + \dfrac{(y+1)^2}{4} = 1$

The conic section is an ellipse .

The center is $P(1, -1)$.

$a^2 = 1 \implies a = 1$

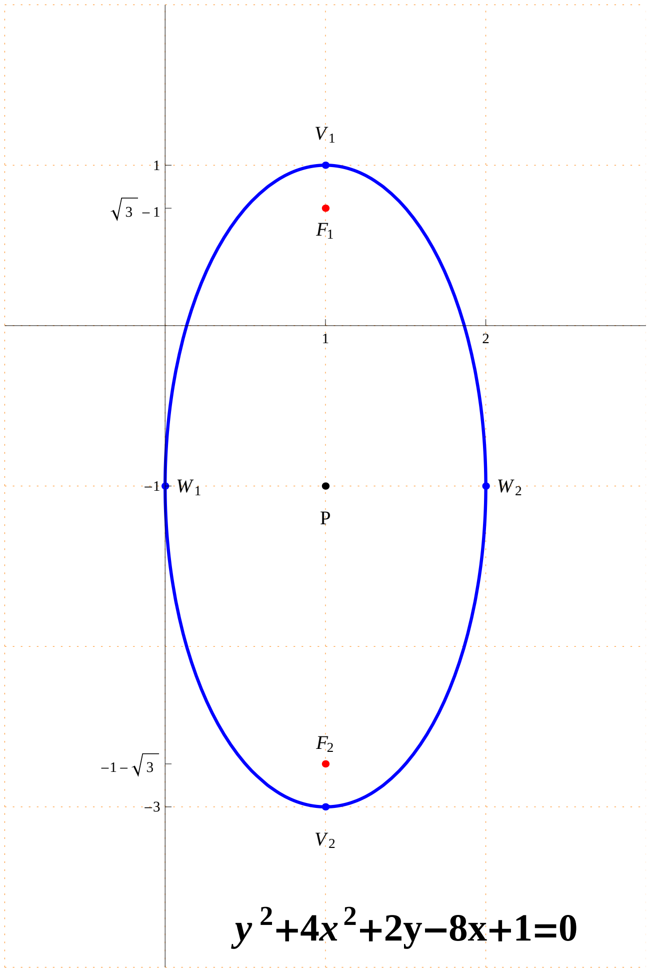
$b^2 = 4 \implies b = 2$

$c^2 = b^2 - a^2 = 4 - 1 = 3 \implies c = \sqrt{3}$

The vertices are $V_1(1, 1)$ and $V_2(1, -3)$

The foci are $F_1\left(1, -1 + \sqrt{3}\right)$ and $F_2\left(1, -1 - \sqrt{3}\right)$

The end-points of the minor axis are $W_1(0, -1)$ and $W_2(2, -1)$



Q.5 Find the standard equation of the parabola with focus $F(-4, 0)$ and with directrix $x = 0$, and then sketch it.

Solution :

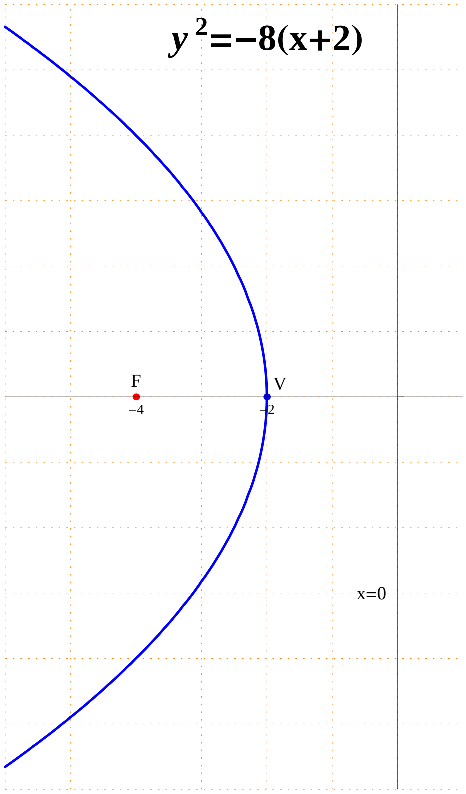
Note that the directrix $x = 0$ is the y-axis and the focus is $F(-4, 0)$ hence the parabola opens to the left.

The parabola has the form $(y - k)^2 = -4a(x - h)$.

The vertex is the midpoint between $F(-4, 0)$ and the directrix $x = 0$, hence $V(-2, 0)$

a is the distance between $F(-4, 0)$ and $V(-2, 0)$, hence $a = 2$

The standard equation of the parabola is $(y - 0)^2 = -8(x + 2)$


$y^2=-8(x+2)$
F
−4
V
−2
x=0

M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[5]

Solution of the Second Mid-Term Exam

Second semester 1436-1437 H

Q.1 Compute the integrals :

(a) $\displaystyle\int \frac{x+2}{(x-2)(x-3)}\ dx$

(b) $\displaystyle\int 2x\cos(x^2+1)\ dx$

(c) $\displaystyle\int x^4 \ln|x|\ dx$

(d) $\displaystyle\int \frac{x+1}{(x-2)^2}\ dx$

(e) $\displaystyle\int \frac{2x}{x^2+1}\ dx$

Solution :

(a) $\displaystyle\int \frac{x+2}{(x-2)(x-3)}\ dx$

Using the method of partial fractions :

$$\frac{x+2}{(x-2)(x-3)} = \frac{A_1}{(x-2)} + \frac{A_2}{x-3}$$

$x+2 = A_1(x-3) + A_2(x-2)$

Put $x=2 :\ 2+2 = A_1(2-3) \implies 4 = -A_1 \implies A_1 = -4$

Put $x=3 :\ 3+2 = A_2(3-2) \implies A_2 = 5$

$$\int \frac{x+2}{(x-2)(x-3)}\ dx = \int \left(\frac{-4}{x-2} + \frac{5}{x-3}\right)\ dx$$

$$= -4\int \frac{1}{x-2}\ dx + 5\int \frac{1}{x-3}\ dx = -4\ln|x-2| + 5\ln|x-3| + c$$

(b) $\displaystyle\int 2x\cos(x^2+1)\ dx$

Using the formula $\displaystyle\int \cos(f(x))\, f'(x)\ dx = \sin(f(x)) + c$

$$\int 2x\cos(x^2+1)\ dx = \sin(x^2+1) + c$$

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(c) $\int x^4 \ln|x| \; dx$

Using integration by parts :

$$u = \ln|x| \qquad dv = x^4 dx$$
$$du = \frac{1}{x} \; dx \qquad v = \frac{x^5}{5}$$

$$\int x^4 \ln|x| \; dx = \frac{x^5}{5} \ln|x| - \int \frac{x^5}{5} \; \frac{1}{x} \; dx = \frac{x^5}{5} \ln|x| - \frac{1}{5} \int x^4 \; dx$$

$$= \frac{x^5}{5} \ln|x| - \frac{1}{5} \; \frac{x^5}{5} + c$$

(d) $\int \frac{x+1}{(x-2)^2} \; dx$

Using the method of partial fractions :

$$\frac{x+1}{(x-2)^2} = \frac{A_1}{x-2} + \frac{A_2}{(x-2)^2}$$

$$x + 1 = A_1(x-2) + A_2 = A_1 x - 2A_1 + A_2$$

By comparing the coefficients of both sides :

$A_1 = 1$

$-2A_1 + A_2 = 1 \implies -2 + A_2 = 1 \implies A_2 = 3$

$$\int \frac{x+1}{(x-2)^2} \; dx = \int \left(\frac{1}{x-2} + \frac{3}{(x-2)^2} \right) \; dx$$

$$= \int \frac{1}{x-2} \; dx + 3 \int (x-2)^{-2} \; dx = \ln|x-2| + 3 \; \frac{(x-2)^{-1}}{-1} + c$$

(e) $\int \frac{2x}{x^2+1} \; dx$

Using the formula $\int \frac{f'(x)}{f(x)} \; dx = \ln|f(x)| + c$

$$\int \frac{2x}{x^2+1} \; dx = \ln(x^2+1) + c$$

Q.2

(a) Sketch the region **R** determined by the curves :

$y = x^2$, $y = -x + 6$ and $y = 0$

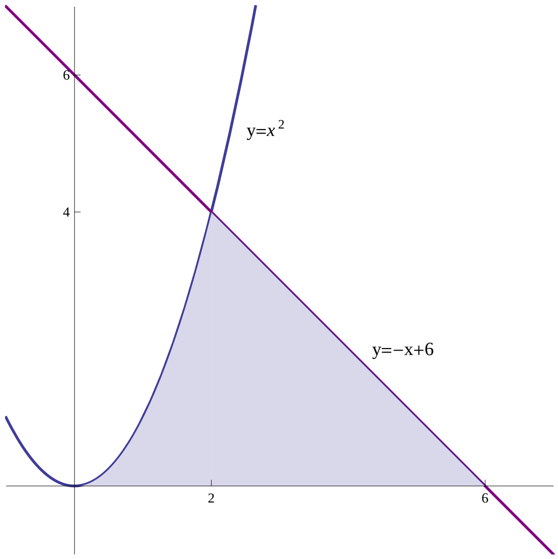
(b) Find the area of the region **R** described in part **(a)**.

Solution :

(a) $y = 0$ is the x-axis .

$y = -x + 6$ is a straight line passes through $(0, 6)$ with slope -1.

$y = x^2$ is a parabola opens upwards with vertex $(0, 0)$



(b) **First solution :**

Points of intersection of $y = x^2$ and $y = -x + 6$:

$x^2 = -x + 6 \implies x^2 + x - 6 = 0 \implies (x - 2)(x + 3) = 0$

$\implies x = 2$, $x = -3$, in this case $y = 4$, $y = 9$

$y = -x + 6 \implies x = -y + 6$

$y = x^2 \implies x = \sqrt{y}$

Area $= \int_0^4 [(-y + 6) - \sqrt{y}\ ]\ dy = \int_0^4 (-y - \sqrt{y} + 6)\ dy$

$= \left[-\frac{y^2}{2} - \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + 6y \right]_0^4$

$= \left(-\frac{4^2}{2} - \frac{2}{3}\ (4)^{\frac{3}{2}} + 6 \times 4 \right) - \left(-\frac{0^2}{2} - \frac{2}{3}\ (0)^{\frac{3}{2}} + 6 \times 0 \right)$

$= -8 - \frac{16}{3} + 24 = 16 - \frac{16}{3} = \frac{48 - 16}{3} = \frac{32}{3}$

Second solution :

Points of intersection of $y = x^2$ and $y = -x + 6$:

$x^2 = -x + 6 \implies x^2 + x - 6 = 0 \implies (x - 2)(x + 3) = 0$

$\implies x = 2 \ , \ x = -3 \ .$

Point of intersection of $y = -x + 6$ and $y = 0$:

$-x + 6 = 0 \implies x = 6$

$$\text{Area} = \int_0^2 x^2 \ dx + \int_2^6 (-x + 6) \ dx$$

$$= \left[\frac{x^3}{3}\right]_0^2 + \left[-\frac{x^2}{2} + 6x\right]_2^6$$

$$= \left[\frac{2^3}{3} - \frac{0^3}{3}\right] + \left[\left(-\frac{6^2}{2} + 6 \times 6\right) - \left(-\frac{2^2}{2} + 6 \times 2\right)\right]$$

$$= \left(\frac{8}{3} - 0\right) + [(-18 + 36) - (-2 + 12)] = \frac{8}{3} + 8 = \frac{8 + 24}{3} = \frac{32}{3}$$

Q.3

(a) Sketch the region **R** determined by the curves :

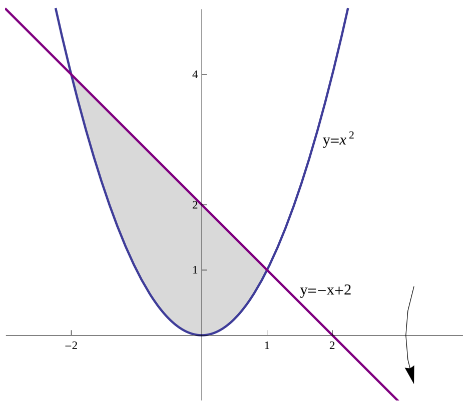
$x = x^2$ and $y = -x + 2$

(b) Find the volume of the solid generated by rotating the region **R** in part **(a)** about the x-axis.

Solution :

(a) $y = -x + 2$ is a straight line passes through $(0, 2)$ with slope -1.

$y = x^2$ is a parabola opens upwards with vertex $(0, 0)$



(b) Points of intersection of $y = x^2$ and $y = -x + 2$:

$x^2 = -x + 2 \implies x^2 + x - 2 = 0 \implies (x - 1)(x + 2) = 0$

$\implies x = 1 \ , \ x = -2 \ .$

Using Washer method :

$$\text{Volume} = \pi \int_{-2}^{1} \left[(-x+2)^2 - \left(x^2\right)^2 \right] \ dx = \pi \int_{-2}^{1} \left(x^2 - 4x + 4 - x^4\right) \ dx$$

$$= \pi \int_{-2}^{1} \left(-x^4 + x^2 - 4x + 4\right) \ dx = \left[-\frac{x^5}{5} + \frac{x^3}{3} - 2x^2 + 4x \right]_{-2}^{1}$$

$$= \pi \left[\left(-\frac{1^5}{5} + \frac{1^3}{3} - 2(1^2) + 4 \times 1 \right) - \left(-\frac{(-2)^5}{5} + \frac{(-2)^3}{3} - 2((-2)^2) + 4 \times -2 \right) \right]$$

$$= \pi \left[\left(-\frac{1}{5} + \frac{1}{3} - 2 + 4 \right) - \left(\frac{32}{5} - \frac{8}{3} - 8 - 8 \right) \right]$$

$$= \pi \left(-\frac{1}{5} + \frac{1}{3} + 2 - \frac{32}{5} + \frac{8}{3} + 16 \right)$$

$$= \pi \left(21 - \frac{33}{5} \right) = \pi \left(\frac{105 - 33}{5} \right) = \frac{72}{5} \pi$$

M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[6]

Solution of the Final Exam

Second semester 1436-1437 H

Q.1 **(a)** Compute **AB** for $\mathbf{A}=\begin{pmatrix}0&2&1\\3&2&3\\3&2&1\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}1&2\\0&2\\3&4\end{pmatrix}$

(b) Compute the determinant $\begin{vmatrix}1&2&0\\0&3&1\\2&0&0\end{vmatrix}$.

(c) Solve by Gauss-Jordan Mehod: $\begin{array}{rcrcrcr} x&-&2y&+&z&=&0\\ x&-&3y&-&z&=&-3\\ 2x&-&y&-&2z&=&-1\end{array}$

Solution :

(a) $\mathbf{AB}=\begin{pmatrix}0&2&1\\3&2&3\\3&2&1\end{pmatrix}\begin{pmatrix}1&2\\0&2\\3&4\end{pmatrix}$

$$=\begin{pmatrix}0+0+3&0+4+4\\3+0+9&6+4+12\\3+0+3&6+4+4\end{pmatrix}=\begin{pmatrix}3&8\\12&22\\6&14\end{pmatrix}$$

(b) **Solution (1):** Using Sarrus Method

$$\begin{array}{ccccc}1&2&0&1&2\\0&3&1&0&3\\2&0&0&2&0\end{array}$$

$$\begin{vmatrix}1&2&0\\0&3&1\\2&0&0\end{vmatrix}=(0+4+0)-(0+0+0)=4-0=4$$

Solution (2): $\begin{vmatrix}1&2&0\\0&3&1\\2&0&0\end{vmatrix}\xrightarrow{C_1\longleftrightarrow C_3}-1\times\begin{vmatrix}0&2&1\\1&3&0\\0&0&2\end{vmatrix}$

$$\xrightarrow{R_1\longleftrightarrow R_2}-1\times-1\times\begin{vmatrix}1&3&0\\0&2&1\\0&0&2\end{vmatrix}=-1\times-1\times(1\times2\times2)=4$$

Solution (3) : Using the definition (using the third row) :

$$\begin{vmatrix}1&2&0\\0&3&1\\2&0&0\end{vmatrix}=2\times\begin{vmatrix}2&0\\3&1\end{vmatrix}-0\times\begin{vmatrix}1&0\\0&1\end{vmatrix}+0\times\begin{vmatrix}1&2\\0&3\end{vmatrix}$$

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$= 2(2-0) - 0 + 0 = 4$

(c) Using Gauss-Jordan Mehod :

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 1 & -3 & -1 & -3 \\ 2 & -1 & -2 & -1 \end{array}\right) \xrightarrow{-R_1+R_2} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -1 & -2 & -3 \\ 2 & -1 & -2 & -1 \end{array}\right)$$

$$\xrightarrow{-2R_1+R_3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -1 & -2 & -3 \\ 0 & 3 & -4 & -1 \end{array}\right) \xrightarrow{3R_2+R_3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & -10 & -10 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{10}R_3} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{2R_3+R_2} \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{-R_3+R_1} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{-1\times R_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{2R_2+R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

The solution is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Q.2 **(a)** Find the standard equation of the parabola with focus $F(2,4)$ and vertex $V(2,3)$, and then sketch it.

(b) Find The elements of the conic section $x^2 + 4y^2 - 16y - 2x + 1 = 0$ and sketch it.

Solution :

(a) The focus is upper than the vertex , hence the parabola opens upwards.

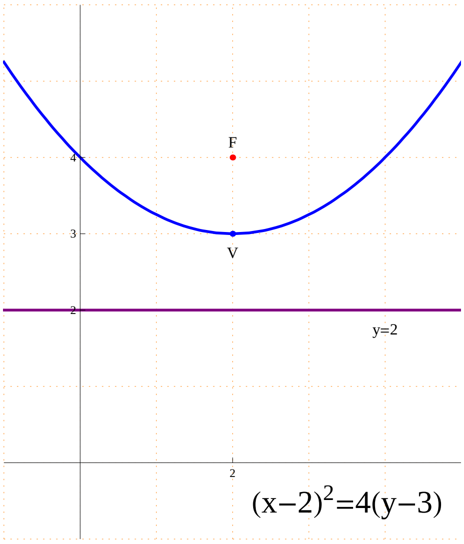
The standard equation of the parabola is $(x-h)^2 = 4a(y-k)$

The vertex is $V(2,3)$, hence $h = 2$ and $k = 3$

a is the distance between V and F , hence $a = 1$

The standard equation of the parabola is $(x-2)^2 = 4(y-3)$

The directrix is $y = 2$



(b) $x^2 + 4y^2 - 16y - 2x + 1 = 0$

$x^2 - 2x + 4y^2 - 16y = -1$

$x^2 - 2x + 4(y^2 - 4y) = -1$

By completing the square

$(x^2 - 2x + 1) + 4(y^2 - 4y + 4) = -1 + 1 + 16$

$(x-1)^2 + 4(y-2)^2 = 16$

$$\frac{(x-1)^2}{16} + \frac{4(y-2)^2}{16} = 1$$

$$\frac{(x-1)^2}{16} + \frac{(y-2)^2}{4} = 1$$

The conic section is an ellipse.

The center is $P(1, 2)$.

$a^2 = 16 \implies a = 4.$

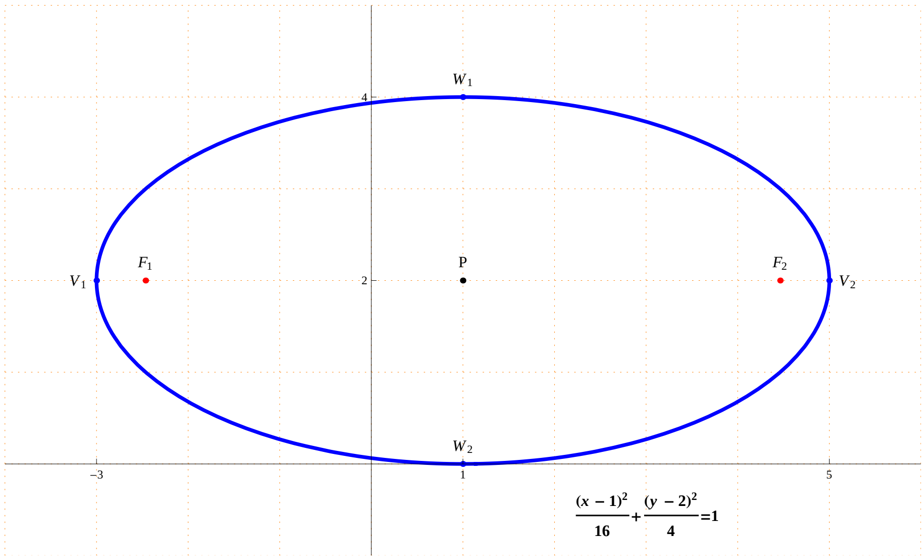
$b^2 = 4 \implies b = 2.$

$c^2 = a^2 - b^2 = 16 - 4 = 12 \implies c = \sqrt{12}.$

The vertices are $V_1(-3, 2)$ and $V_2(5, 2)$

The foci are $F_1\left(1 - \sqrt{12}, 2\right)$ and $F_2\left(1 + \sqrt{12}, 2\right)$.

The end-points of the minor axis are $W_1(1, 4)$ and $W_2(1, 0)$.



Q.3 **(a)** Compute the integrals :

(i) $\int 2x(x^2+6)^5\ dx$ (ii) $\int x\cos x\ dx$ (iii) $\int \frac{1}{(x+1)(x-2)}\ dx$

(b) Find the area of the region bounded by the graphs :

$y = 3$ and $y = x^2 - 1$.

(c) The region R between the curves $y = x^2$ and $y = \sqrt{x}$ is rotated about the x-axis to form a solid of revolution S . Find the volume of S.

(d) Using polar coordinates find the area of the circle with polar equation $r = 2$.

Solution :

(a) (i) $\int 2x(x^2+6)^5\ dx = \frac{(x^2+6)^6}{6} + c$

Using the formula $\int [f(x)]^n\, f'(x)\ dx = \frac{[f(x)]^{n+1}}{n+1} + c$ where $n \neq -1$.

(ii) $\int x\cos x\ dx$

Using integration by parts

$$\begin{array}{ll} u = x & dv = \cos x\ dx \\ du = dx & v = \sin x \end{array}$$

$$\int x\cos x\ dx = x\sin x - \int \sin x\ dx$$

$= x \sin x - (-\cos x) + c = x \sin x + \cos x + c$

(iii) $\displaystyle\int \frac{1}{(x+1)(x-2)}\ dx$

Using the method of partial fractions

$$\frac{1}{(x+1)(x-2)} = \frac{A_1}{x+1} + \frac{A_2}{x-2}$$

$1 = A_1(x-2) + A_2(x+1)$

Put $x = -1$ then $1 = -3A_1 \implies A_1 = -\dfrac{1}{3}$

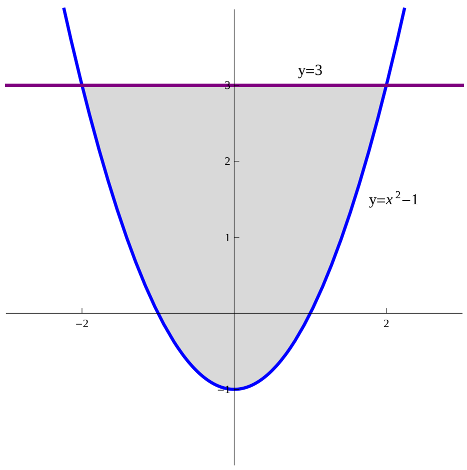
Put $x = 2$ then $1 = 3A_2 \implies A_1 = \dfrac{1}{3}$

$$\int \frac{1}{(x+1)(x-2)}\ dx = \int \left(\frac{-\frac{1}{3}}{x+1} + \frac{\frac{1}{3}}{x-2} \right)\ dx$$

$$= -\frac{1}{3} \int \frac{1}{x+1}\ dx + \frac{1}{3} \int \frac{1}{x-2}\ dx = -\frac{1}{3} \ln|x+1| + \frac{1}{3} \ln|x-2| + c$$

(b) $y = 3$ is a straight line parallel to the x-axis and passes through $(0,3)$.

$y = x^2 - 1 \implies y + 1 = x^2$ is a parabola with vertex $(-1, 0)$ and opens upwards.



Points of intersection of $y = 3$ and $y = x^2 - 1$:

$x^2 - 1 = 3 \implies x^2 - 4 = 0 \implies (x-2)(x+2) = 0 \implies x = -2\ ,\ x = 2$

$$\text{Area} = \int_{-2}^{2} \left[3 - (x^2 - 1)\right]\ dx = \int_{-2}^{2} (4 - x^2)\ dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$= \left(4 \times 2 - \frac{2^3}{3}\right) - \left(4 \times -2 - \frac{(-2)^3}{3}\right) = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = 16 - \frac{16}{3} = \frac{48-16}{3} = \frac{32}{3}$$

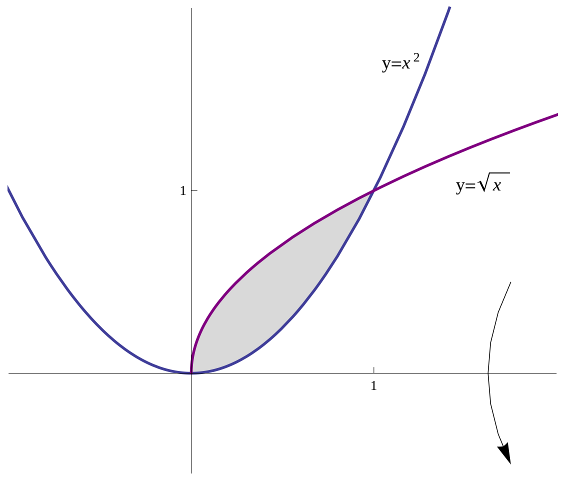
(c) $y = \sqrt{x}$ is the upper-half of the parabola $y^2 = x$ with vertex $(0,0)$ and opens to the right.

$y = x^2$ is a parabola opens upwards with vertex $(0,0)$.

Points of intersection of $y = x^2$ and $y = \sqrt{x}$:

$x^2 = \sqrt{x} \implies x^4 = x \implies x^4 - x = 0$

$\implies x(x^3-1) = 0 \implies x = 0\ ,\ x^3 = 1 \implies x = 0\ ,\ x = 1$



Using Washer Method :

$$\text{Volume} = \pi \int_0^1 \left[(\sqrt{x})^2 - (x^2)^2\right]\ dx = \pi \int_0^1 (x - x^4)\ dx$$

$$= \pi\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = \pi\left[\left(\frac{1}{2} - \frac{1}{5}\right) - (0-0)\right]$$

$$= \pi\left(\frac{5-2}{10}\right) = \frac{3}{10}\pi$$

(d) $r = 2$ is a circle with center $(0,0)$ and radius equals 2.

$$\text{Area} = \frac{1}{2}\int_0^{2\pi} (2)^2\ d\theta = \frac{1}{2}\int_0^{2\pi} 4\ d\theta = 4 \times \frac{1}{2}\int_0^{2\pi} 1\ d\theta$$

$$= 2\,[\theta]_0^{2\pi} = 2\,(2\pi - 0) = 4\pi$$

Q.4 **(a)** Find f_x and f_y for the function $f(x,y) = x^2y^6 + xye^x + \ln(x+y)$.

(b) Solve the differential equation $x\,\dfrac{dy}{dx} + y = e^x$.

Solution :

(a) $f_x = (2x)y^6 + y\,[1 \times e^x + xe^x] + \dfrac{1+0}{x+y}$

$= 2xy^6 + ye^x + xye^x + \dfrac{1}{x+y}$

$f_y = x^2(6y^5) + xe^x \times 1 + \dfrac{0+1}{x+y}$

$= 6x^2y^5 + xe^x + \dfrac{1}{x+y}$

(b) $x\,\dfrac{dy}{dx} + y = e^x$

$\dfrac{dy}{dx} + \dfrac{1}{x}y = \dfrac{1}{x}e^x$

It is a First-order differential equation .

$P(x) = \dfrac{1}{x}$ and $Q(x) = \dfrac{1}{x}e^x$

The integrating factor is :

$u(x) = e^{\int P(x)\ dx} = e^{\int \frac{1}{x}\ dx} = e^{\ln|x|} = x$

The general solution of the differential equation is :

$y = \dfrac{1}{u(x)} \displaystyle\int u(x)Q(x)\ dx = \dfrac{1}{x}\int x\ \dfrac{1}{x}e^x\ dx = \dfrac{1}{x}\int e^x\ dx$

$= \dfrac{1}{x}\,(e^x + c) = \dfrac{e^x}{x} + \dfrac{c}{x}$