M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[1]

Solution of the First Mid-Term Exam

First semester 1435-1436 H

Q.1 Let $\mathbf{A} = \begin{pmatrix} 2 & 4 & 0 \\ 1 & 3 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 1 \end{pmatrix}$

Compute (if possible) : **AB** and **BA**

Solution :

$$\mathbf{AB} = \begin{pmatrix} 2 & 4 & 0 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+12+0 & 4+4+0 \\ 1+9+0 & 2+3+1 \end{pmatrix} = \begin{pmatrix} 14 & 8 \\ 10 & 6 \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 4 & 0 \\ 1 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+2 & 4+6 & 0+2 \\ 6+1 & 12+3 & 0+1 \\ 0+1 & 0+3 & 0+1 \end{pmatrix} = \begin{pmatrix} 4 & 10 & 2 \\ 7 & 15 & 1 \\ 1 & 3 & 1 \end{pmatrix}$$

Q.2 Compute The determinant $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$

Solution (1) : Using Sarrus Method

$$\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ 3 & 2 & 1 & 3 & 2 \\ 2 & 1 & 3 & 2 & 1 \end{matrix}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = (6+4+9) - (12+1+18) = 19 - 31 = -12$$

Solution (2) : Using The definition

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 1 \times \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} - 2 \times \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} + 3 \times \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (6-1) - 2(9-2) + 3(3-4) = 5 - 14 - 3 = 5 - 17 = -12$$

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Q.3 Find all the elements of the conic section $x^2 = 2y + 2x$ and sketch it.

Solution :

$x^2 - 2x = 2y$

$x^2 - 2x + 1 = 2y + 1$

$(x-1)^2 = 2\left(y + \frac{1}{2}\right)$

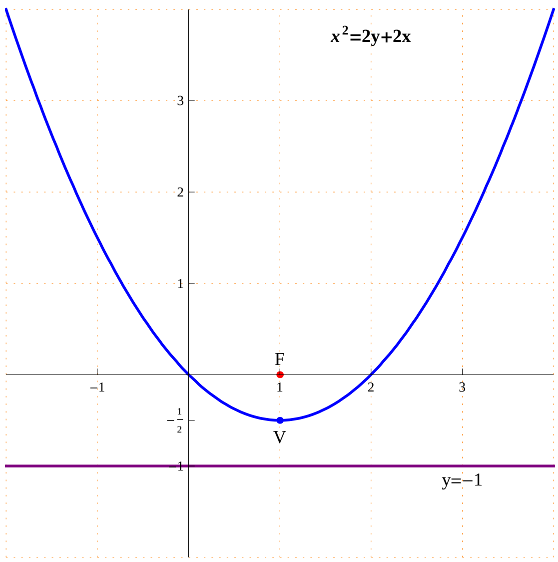
The conic section is a parabola opens upwards.

The vertex is $V\left(1, -\frac{1}{2}\right)$.

$4a = 2 \implies a = \frac{2}{4} = \frac{1}{2}$.

The focus is $F\left(1, -\frac{1}{2} + \frac{1}{2}\right) = (1, 0)$.

The equation of the directrix is $y = -\frac{1}{2} - \frac{1}{2} \implies y = -1$



Q.4 Find the standard equation of the ellipse with vertex $(1, 2)$ and with foci $(2, 2)$ and $(10, 2)$, and then sketch it.

Solution :

The equation of the ellipse has the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

The center of the ellipse is the mid-point of the two foci .

The center is $P\left(\frac{2+10}{2}, \frac{2+2}{2}\right) = (6,2)$, hence $h=6$ and $k=2$.

The two foci lie on a line parallel to the x-axis , hence $a > b$

c is the distance between the center and one of the foci , hence $c = 4$

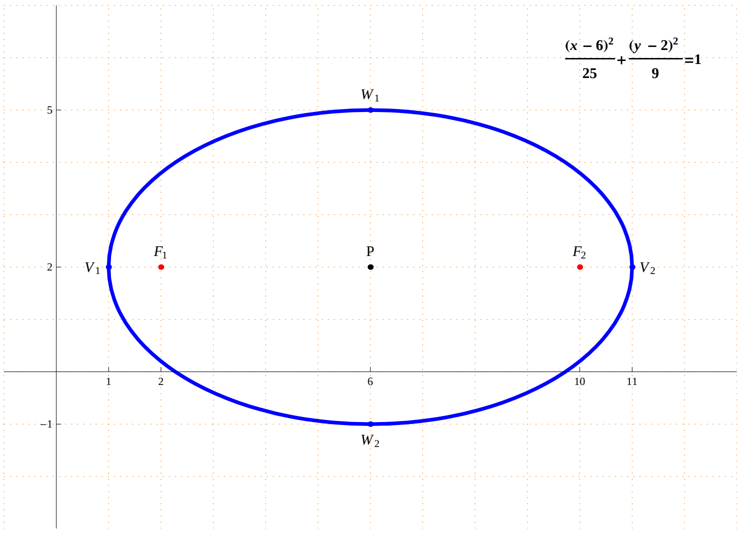
a is the distance between the center $(6,2)$ and the vertex $V_1(1,2)$, hence $a = 5$.

$c^2 = a^2 - b^2 \implies 4^2 = 5^2 - b^2 \implies b^2 = 25 - 16 = 9 \implies b = 3$.

The equation of the ellipse is $\frac{(x-6)^2}{25} + \frac{(y-2)^2}{9} = 1$.

The other vertex is $V_2(11,2)$.

The end-points of the minor axis are $W_1(6,5)$ and $W_2(6,-1)$



Q.5 Solve by Cramer's Rule the following linear system $\begin{cases} 2x - 3y = 3 \\ x + y = 4 \end{cases}$

Solution :

$\mathbf{A} = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$, $\mathbf{A}_x = \begin{pmatrix} 3 & -3 \\ 4 & 1 \end{pmatrix}$, $\mathbf{A}_y = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

$|\mathbf{A}| = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = (2 \times 1) - (-3 \times 1) = 2 - (-3) = 5$

$|\mathbf{A}_x| = \begin{vmatrix} 3 & -3 \\ 4 & 1 \end{vmatrix} = (3 \times 1) - (-3 \times 4) = 3 - (-12) = 15$

$|\mathbf{A}_y| = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = (2 \times 4) - (3 \times 1) = 8 - 3 = 5$

$x = \frac{|\mathbf{A}_x|}{|\mathbf{A}|} = \frac{15}{5} = 3$

$y = \frac{|\mathbf{A}_y|}{|\mathbf{A}|} = \frac{5}{5} = 1$

The solution of the linear system is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

M 104 - GENERAL MATHEMATICS -2-

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Solution of the Second Mid-Term Exam

First semester 1435-1436 H

Q.1 Solve by Gauss Elimination method the linear system :

$$\begin{cases} x & + & 3y & + & 2z & = & 2 \\ 2x & - & y & - & 3z & = & -3 \\ 3x & - & 4y & - & z & = & 5 \end{cases}$$

Solution : The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 2 & -1 & -3 & -3 \\ 3 & -4 & -1 & 5 \end{array}\right] \xrightarrow{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & -7 & -7 & -7 \\ 3 & -4 & -1 & 5 \end{array}\right]$$

$$\xrightarrow{-3R_1+R_3} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & -7 & -7 & -7 \\ 0 & -13 & -7 & -1 \end{array}\right] \xrightarrow{-\frac{1}{7}R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -13 & -7 & -1 \end{array}\right]$$

$$\xrightarrow{13R_2+R_3} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 6 & 12 \end{array}\right]$$

$6z = 12 \implies z = 2$

$y + z = 1 \implies y + 2 = 1 \implies y = -1$

$x + 3y + 2z = 2 \implies x - 3 + 4 = 2 \implies x + 1 = 2 \implies x = 1$

The solution of the linear system is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

Q.2 Compute the integrals :

(a) $\displaystyle\int_0^1 xe^x \, dx$

(b) $\displaystyle\int \frac{x+1}{(x+2)(x-3)} \, dx$

(c) $\displaystyle\int 4x(2x^2+3)^3 \, dx$

(d) $\displaystyle\int x \, \ln x \, dx$

(e) $\displaystyle\int \frac{x}{(x-1)^2} \, dx$

Solution :

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(a) Using integration by parts :

$$u = x \qquad dv = e^x \ dx$$
$$du = dx \qquad v = e^x$$

$$\int_0^1 xe^x \ dx = [xe^x]_0^1 - \int_0^1 e^x \ dx = [xe^x]_0^1 - [e^x]_0^1$$
$$= \left[\left(1 \ e^1\right) - \left(0 \ e^0\right)\right] - \left[e^1 - e^0\right] = (e - 0) - (e - 1) = e - e + 1 = 1$$

(b) Using the method of partial fractions :

$$\frac{x+1}{(x+2)(x-3)} = \frac{A_1}{x+2} + \frac{A_2}{x-3}$$

$$x + 1 = A_1(x-3) + A_2(x+2)$$

Put $x = -2 : \ -2 + 1 = A_1(-2-3) \implies -1 = -5A_1 \implies A_1 = \dfrac{1}{5}$

Put $x = 3 : \ 3 + 1 = A_2(3+2) \implies 4 = 5A_2 \implies A_2 = \dfrac{4}{5}$

$$\int \frac{x+1}{(x+2)(x-3)} \ dx = \int \left(\frac{\frac{1}{5}}{x+2} + \frac{\frac{4}{5}}{x-3}\right) \ dx$$
$$= \frac{1}{5}\int \frac{1}{x+2} \ dx + \frac{4}{5}\int \frac{1}{x-3} \ dx = \frac{1}{5}\ln|x+2| + \frac{4}{5}\ln|x-3| + c$$

(c) Using the formula $\displaystyle\int [f(x)]^n f'(x) \ dx = \frac{[f(x)]^{n+1}}{n+1} + c$, where $n \neq -1$

$$\int 4x(2x^2+3)^3 \ dx = \int (2x^2+3)^3 4x \ dx = \frac{(2x^2+3)^4}{4} + c$$

(d) Using integration by parts :

$$u = \ln x \qquad dv = x \ dx$$
$$du = \frac{1}{x} \ dx \qquad v = \frac{x^2}{2}$$

$$\int x \ \ln x \ dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2} \ \frac{1}{x} \ dx$$
$$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x \ dx = \frac{x^2}{2}\ln x - \frac{1}{2}\frac{x^2}{2} + c$$

(e) Using the method of partial fractions :

$$\frac{x}{(x-1)^2} = \frac{A_1}{(x-1)} + \frac{A_2}{(x-1)^2}$$

$x = A_1(x-1) + A_2 = A_1 x - A_1 + A_2$

Comparing the coefficients of both sides

$A_1 = 1$

$-A_1 + A_2 = 0 \implies -1 + A_2 = 0 \implies A_2 = 1$

$$\int \frac{x}{(x-1)^2}\, dx = \int \left(\frac{1}{x-1} + \frac{1}{(x-1)^2} \right)\, dx$$

$$= \int \frac{1}{x-1}\, dx + \int \frac{1}{(x-1)^2} = \int \frac{1}{x-1}\, dx + \int (x-1)^{-2} dx$$

$$= \ln|x-1| + \frac{(x-1)^{-1}}{-1} + c$$

Q.3 (a) Sketch the region **R** determined by the curves :
$y = x^2$, $x = 1$, $x = 2$ and $y = 0$.

(b) Find the area of the region **R**.

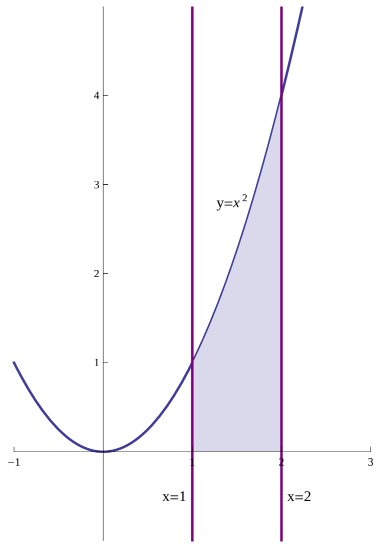
Solution :

(a) $y = x^2$ is a parabola opens upwards with vertex $(0, 0)$.

$x = 1$ is a straight line parallel to the y-axis and passes through $(1, 0)$.

$x = 2$ is a straight line parallel to the y-axis and passes through $(2, 0)$.

$y = 0$ is the x-axis .



(b) $$\text{Area} = \int_1^2 x^2 dx = \left[\frac{x^3}{3} \right]_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[3]

Solution of the Final Exam

First semester 1435-1436 H

Q.1 **(a)** Compute **AB** and **BA** for $\mathbf{A} = \begin{pmatrix} 0 & 2 & 5 \\ -3 & 2 & 3 \\ -3 & 2 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 2 & 1 \\ 3 & -2 & 1 \\ 3 & 2 & 1 \end{pmatrix}$

(b) Compute the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix}$.

(c) Solve by Cramer's rule : $\begin{array}{rcrcr} 4x & - & 2y & = & 2 \\ x & - & 3y & = & -2 \end{array}$

Solution :

(a) $\mathbf{AB} = \begin{pmatrix} 0 & 2 & 5 \\ -3 & 2 & 3 \\ -3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 2 & 1 \\ 3 & -2 & 1 \\ 3 & 2 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 0+6+15 & 0-4+10 & 0+2+5 \\ -9+6+9 & -6-4+6 & -3+2+3 \\ -9+6+3 & -6-4+2 & -3+2+1 \end{pmatrix} = \begin{pmatrix} 21 & 6 & 7 \\ 6 & -4 & 2 \\ 0 & -8 & 0 \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} 3 & 2 & 1 \\ 3 & -2 & 1 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 2 & 5 \\ -3 & 2 & 3 \\ -3 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0-6-3 & 6+4+2 & 15+6+1 \\ 0+6-3 & 6-4+2 & 15-6+1 \\ 0-6-3 & 6+4+2 & 15+6+1 \end{pmatrix} = \begin{pmatrix} -9 & 12 & 22 \\ 3 & 4 & 10 \\ -9 & 12 & 22 \end{pmatrix}$$

(b) **Solution (1):** Using Sarrus Method

$$\begin{array}{ccccc} 1 & 2 & 3 & 1 & 2 \\ 1 & 3 & 2 & 1 & 3 \\ 3 & 2 & 1 & 3 & 2 \end{array}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} = (3+12+6) - (27+4+2) = 21 - 33 = -12$$

Solution (2): $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} \xrightarrow{-R_1+R_2} \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix}$

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$$\xrightarrow{-3R_1+R_3} \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & -4 & -8 \end{vmatrix} \xrightarrow{4R_2+R_3} \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -12 \end{vmatrix} = 1 \times 1 \times -12 = -12$$

Solution (3) : Using the definition of the determinant

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 1 \times \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} - 2 \times \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 3 \times \begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix}$$

$$= 1 \times (3-4) - 2 \times (1-6) + 3 \times (2-9) = -1 + 10 - 21 = -12$$

(c) Using Cramer's rule :

$$\mathbf{A} = \begin{pmatrix} 4 & -2 \\ 1 & -3 \end{pmatrix}, \ \mathbf{A}_x = \begin{pmatrix} 2 & -2 \\ -2 & -3 \end{pmatrix}, \ \mathbf{A}_y = \begin{pmatrix} 4 & 2 \\ 1 & -2 \end{pmatrix}$$

$$|\mathbf{A}| = \begin{vmatrix} 4 & -2 \\ 1 & -3 \end{vmatrix} = (4 \times -3) - (1 \times -2) = -12 - (-2) = -12 + 2 = -10$$

$$|\mathbf{A}_x| = \begin{vmatrix} 2 & -2 \\ -2 & -3 \end{vmatrix} = (2 \times -3) - (-2 \times -2) = -6 - 4 = -10$$

$$|\mathbf{A}_y| = \begin{vmatrix} 4 & 2 \\ 1 & -2 \end{vmatrix} = (4 \times -2) - (1 \times 2) = -8 - 2 = -10$$

$$x = \frac{|\mathbf{A}_x|}{|\mathbf{A}|} = \frac{-10}{-10} = 1$$

$$y = \frac{|\mathbf{A}_y|}{|\mathbf{A}|} = \frac{-10}{-10} = 1$$

The solution of the linear system is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Q.2 **(a)** Find the standard equation of the ellipse with foci $(-2, 3)$ and $(6, 3)$ and the length of its major axis is 10 , and then sketch it.

(b) Find The elements of the conic section $9x^2 - 4y^2 - 16y - 18x - 43 = 0$

Solution :

(a) The standard equation of the ellipse is $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

The center $P = (h, k)$ is located between the two foci.

The center is $P = (h, k) = \left(\dfrac{-2+6}{2}, \dfrac{3+3}{2} \right) = (2, 3)$

The major axis (where the two foci are located) is parallel to the x-axis , hence $a > b$.

The length of the major axis equals 10 means $2a = 10 \implies a = 5$.

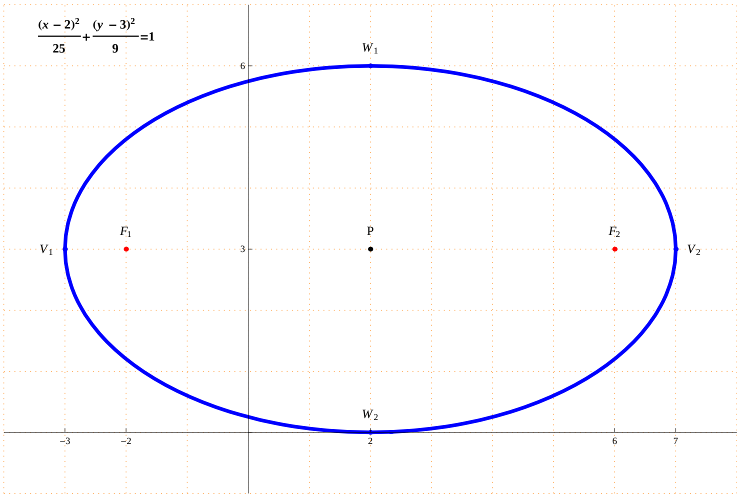
c is the distance between the two foci , hence $c = 4$

$c^2 = a^2 - b^2 \implies 16 = 25 - b^2 \implies b^2 = 25 - 16 = 9 \implies b = 3$

The standard equation of the ellipse is $\dfrac{(x-2)^2}{25} + \dfrac{(y-3)^2}{9} = 1$

The vertices are $V_1 = (-3, 3)$ and $V_2 = (7, 3)$

The end-points of the minor axis are $W_1 = (2, 6)$ and $W_2 = (2, 0)$



(b) $9x^2 - 4y^2 - 16y - 18x - 43 = 0$

$9x^2 - 18x - 4y^2 - 16y = 43$

$9(x^2 - 2x) - 4(y^2 + 4y) = 43$

By completing the square

$9(x^2 - 2x + 1) - 4(y^2 + 4y + 4) = 43 + 9 - 16$

$9(x-1)^2 - 4(y+2)^2 = 36$

$\dfrac{9(x-1)^2}{36} - \dfrac{4(y+2)^2}{36} = 1$

$\dfrac{(x-1)^2}{4} - \dfrac{(y+2)^2}{9} = 1$

The conic section is a hyperbola with transverse axis parallel to the x-axis.

The ceneter is $P = (1, -2)$.

$a^2 = 4 \implies a = 2$ and $b^2 = 9 \implies b = 3$.

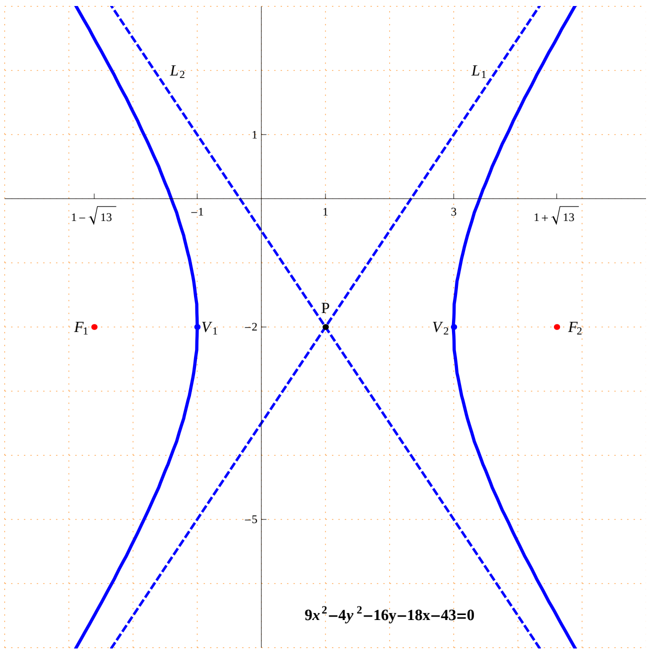
$c^2 = a^2 + b^2 = 9 + 4 = 13 \implies c = \sqrt{13}$.

The vertices are $V_1 = (-1, -2)$ and $V_2 = (3, -2)$.

The foci are $F_1 = \left(1-\sqrt{13}, -2\right)$ and $F_2 = \left(1+\sqrt{13}, -2\right)$.

The equations of the asymptotes are $L_1 \; : \; y+2 = \dfrac{3}{2}\,(x-1)$ and

$L_2 \; : \; y+2 = -\dfrac{3}{2}\,(x-1)$



Q.3 **(a)** Compute the integrals :

(i) $\displaystyle\int \frac{10}{x^2+6x+10}\,dx$ (ii) $\displaystyle\int x^2 \ln x \; dx$ (iii) $\displaystyle\int \frac{x-1}{(x+1)(x-1)^2}\,dx$

(b) Find the area of the surface delimited by the curves :

$y = 0$, $x = 0$, $x = 4$ and $y = \sqrt{x}+5$.

(c) The region R between the curves $y = 0$, $y = x^2$, and $y = -x+2$ is rotated about the y-axis to form a solid of revolution S . Find the volume of S.

Solution :

(a) (i) $\displaystyle\int \frac{10}{x^2+6x+10}\,dx$

Using the formula $\displaystyle\int \frac{f'(x)}{[f(x)]^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right)+c$

$$\int \frac{10}{x^2+6x+10}\,dx = 10\int \frac{1}{(x^2+6x+9)+1}\,dx$$

$$= 10\int \frac{1}{(x+3)^2+1^2}\ dx = 10\tan^{-1}(x+3)+c$$

(ii) $\int x^2 \ln x \ dx$

Using integration by parts

$$u = \ln x \qquad dv = x^2\ dx$$
$$du = \frac{1}{x}dx \qquad v = \frac{x^3}{3}$$

$$\int x^2 \ln x\ dx = \frac{x^3}{3}\ln x - \int \frac{1}{x}\frac{x^3}{3}\ dx$$

$$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\ dx = \frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3} + c$$

(iii) $\displaystyle\int \frac{x-1}{(x+1)(x-1)^2}\ dx = \int \frac{1}{(x-1)(x+1)}\ dx$

Using the method of partial fractions

$$\frac{1}{(x+1)(x-1)} = \frac{A_1}{x+1} + \frac{A_2}{x-1}$$

$$\frac{1}{(x+1)(x-1)} = \frac{A_1(x-1)}{(x+1)(x-1)} + \frac{A_2(x+1)}{(x+1)(x-1)}$$

$$1 = A_1(x-1) + A_2(x+1)$$

Put $x = -1$ then $1 = -2A_1 \implies A_1 = -\frac{1}{2}$

Put $x = 1$ then $1 = 2A_2 \implies A_2 = \frac{1}{2}$

$$\int \frac{1}{(x+1)(x-1)}\ dx = \int \left(\frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}\right)\ dx$$

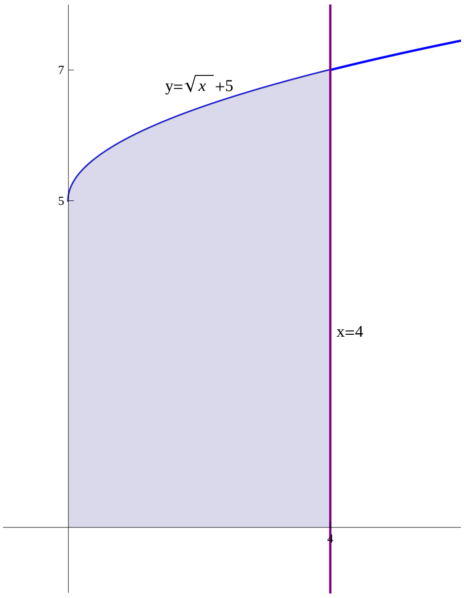
$$= -\frac{1}{2}\int \frac{1}{x+1}\ dx + \frac{1}{2}\int \frac{1}{x-1}\ dx = -\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + c$$

(b) $y = 0$ is the x-axis.

$x = 0$ is the y-axis.

$x = 4$ is a straight line parallel to the y-axis and passes through $(4, 0)$.

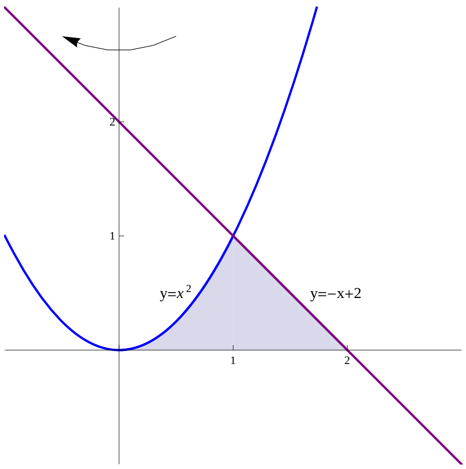
$y = \sqrt{x}+5 \implies (y-5)^2 = x$ is the upper-half of the parabola with vertex $(0, 5)$ and opens to the right



Area $= \int_0^4 \left(\sqrt{x}+5\right)\ dx = \left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + 5x\right]_0^4 = \left[\dfrac{2}{3}\ x^{\frac{3}{2}} + 5x\right]_0^4$

$= \left(\dfrac{2}{3}(4)^{\frac{3}{2}} + 5\times 4\right) - \left(\dfrac{2}{3}(0)^{\frac{3}{2}} + 5\times 0\right) = \dfrac{2}{3}\ (8) + 20 = \dfrac{16}{3} + 20 = \dfrac{76}{3}$

(c)



$y = 0$ is the x-axis.

$y = -x + 2$ is a straight line passes through $(0, 2)$ and its slope is -1.

$y = x^2$ is a parabola opens upwards with vertex $(0, 0)$.

Points of intersection of $y = x^2$ and $y = -x + 2$:

$x^2 = -x+2 \Rightarrow x^2+x-2=0 \Rightarrow (x+2)(x-1)=0 \Rightarrow x=1\ ,\ x=-2$

Points of intersection are $(-2,4)$ and $(1,1)$.

Using Washer Method :

In this case $y=-x+2 \implies x=-y+2$ and $y=x^2 \implies x=\sqrt{y}$

$$\text{Volume} = \pi \int_0^1 \left((-y+2)^2 - (\sqrt{y})^2\right)\ dy = \pi \int_0^1 (4-4y+y^2-y)\ dy$$

$$= \pi \int_0^1 (y^2-5y+4)\ dy = \pi \left[\frac{y^3}{3} - 5\frac{y^2}{2} + 4y\right]_0^1$$

$$= \pi \left[\left(\frac{1}{3} - \frac{5}{2} + 4\right) - (0-0+0)\right] = \pi \left(\frac{2-15+24}{6}\right) = \frac{11}{6}\pi$$

Q.4 **(a)** Find f_x and f_y for the function $f(x,y) = x^2y^3 + ye^x + \dfrac{x}{x+y}$

(b) Solve the differential equation $x\dfrac{dy}{dx} - y = x^2e^x$

Solution :

(a) $f_x = \dfrac{\partial f}{\partial x} = 2xy^3 + ye^x + \dfrac{1(x+y) - x(1+0)}{(x+y)^2} = 2xy^3 + ye^x + \dfrac{y}{(x+y)^2}$

$f_y = 3x^2y^2 + e^x + \dfrac{0(x+y) - x(0+1)}{(x+y)^2} = 3x^2y^2 + e^x - \dfrac{x}{(x+y)^2}$

(b) $x\dfrac{dy}{dx} - y = x^2e^x$

$xy' - y = x^2e^x$

$y' - \dfrac{1}{x}y = xe^x$

It is a first-order linear differential equation.

$P(x) = -\dfrac{1}{x}$ and $Q(x) = xe^x$

The integrating factor is:

$u(x) = ^{\int P(x)\ dx} = e^{\int -\frac{1}{x}\ dx} = e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} = \dfrac{1}{x}$

The general solution of the differential equation is :

$$y = \frac{1}{u(x)} \int u(x)\ Q(x)\ dx = \frac{1}{x^{-1}} \int x^{-1}xe^x dx = x \int e^x\ dx$$

$$= x\left(e^x + c\right) = xe^x + cx$$

M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[4]

Solution of the First Mid-Term Exam

Second semester 1435-1436 H

Q.1 Let $\mathbf{A} = \begin{pmatrix} 2 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 4 \end{pmatrix}$

Compute (if possible) : **AB** and **BA**

Solution :

$$\mathbf{AB} = \begin{pmatrix} 2 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2+12+0 & 4+4+0 \\ 1+6+0 & 2+2+4 \end{pmatrix} = \begin{pmatrix} 14 & 8 \\ 7 & 8 \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 2 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+2 & 4+4 & 0+2 \\ 6+1 & 12+2 & 0+1 \\ 0+4 & 0+8 & 0+4 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 2 \\ 7 & 14 & 1 \\ 4 & 8 & 4 \end{pmatrix}$$

Q.2 Compute The determinant $\begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{vmatrix}$

Solution (1) : Using Sarrus Method

$$\begin{matrix} 1 & 2 & 4 & 1 & 2 \\ 3 & 2 & 1 & 3 & 2 \\ 4 & 1 & 3 & 4 & 1 \end{matrix}$$

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{vmatrix} = (6+8+12) - (32+1+18) = 26 - 51 = -25$$

Solution (2) : Using The definition

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{vmatrix} = 1 \times \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} - 2 \times \begin{vmatrix} 3 & 1 \\ 4 & 3 \end{vmatrix} + 4 \times \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= (6-1) - 2(9-4) + 4(3-8) = 5 - 10 - 20 = 5 - 30 = -25$$

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Q.3 Find all the elements of the conic section $y^2 - 4y = 4x + 4$ and sketch it.

Solution : By completing the square

$y^2 - 4y = 4x + 4$

$y^2 - 4y + 4 = 4x + 4 + 4$

$y^2 - 4y + 4 = 4x + 8$

$(y - 2)^2 = 4(x + 2)$

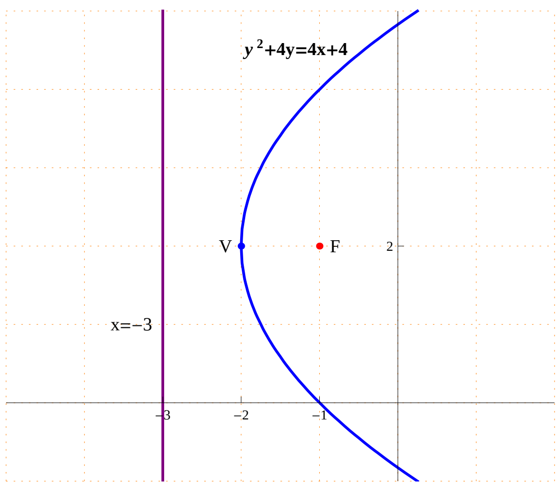
The conic section is a parabola opens to the right.

The vertex is $V(-2, 2)$.

$4a = 4 \implies a = \frac{4}{4} = 1.$

The focus is $F(-2 + 1, 2) = (-1, 2)$.

The equation of the directrix is $x = -2 - 1 = -3$



Q.4 Find the standard equation of the ellipse with vertex $(1, 4)$ and with foci $(2, 4)$ and $(10, 4)$, and then sketch it.

Solution :

The equation of the ellipse has the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

The center of the ellipse is the mid-point of the two foci .

The center is $P\left(\frac{2+10}{2}, \frac{4+4}{2}\right) = (6, 4)$, hence $h = 6$ and $k = 2$.

The two foci lie on a line parallel to the x-axis , hence $a > b$

c is the distance between the center and one of the foci , hence $c = 4$

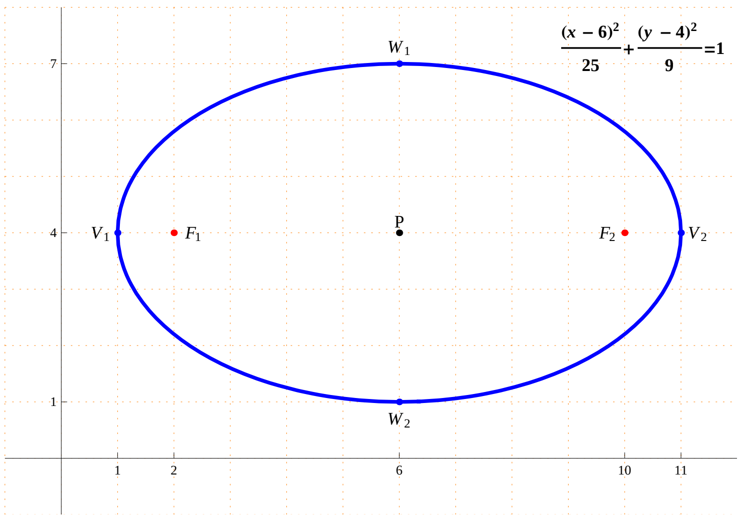
a is the distance between the center $(6, 4)$ and the vertex $V_1(1, 4)$, hence $a = 5$.

$c^2 = a^2 - b^2 \implies 4^2 = 5^2 - b^2 \implies b^2 = 25 - 16 = 9 \implies b = 3$.

The equation of the ellipse is $\dfrac{(x-6)^2}{25} + \dfrac{(y-4)^2}{9} = 1$.

The other vertex is $V_2(11, 4)$.

The end-points of the minor axis are $W_1(6, 7)$ and $W_2(6, 1)$



Q.5 Solve by Cramer's Rule the following linear system $\begin{cases} 4x - 3y = 1 \\ 2x + y = 3 \end{cases}$

Solution :

$\mathbf{A} = \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$, $\mathbf{A}_x = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix}$, $\mathbf{A}_y = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

$|\mathbf{A}| = \begin{vmatrix} 4 & -3 \\ 2 & 1 \end{vmatrix} = (4 \times 1) - (2 \times -3) = 4 - (-6) = 4 + 6 = 10$

$|\mathbf{A}_x| = \begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix} = (1 \times 1) - (3 \times -3) = 1 - (-9) = 1 + 9 = 10$

$|\mathbf{A}_y| = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} = (4 \times 3) - (2 \times 1) = 12 - 2 = 10$

$x = \dfrac{|\mathbf{A}_x|}{|\mathbf{A}|} = \dfrac{10}{10} = 1$

$$y = \frac{|\mathbf{A}_y|}{|\mathbf{A}|} = \frac{10}{10} = 1$$

The solution of the linear system is $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[5]

Solution of the Second Mid-Term Exam

Second semester 1435-1436 H

Q.1 Find the area of the surface bounded by the curves :

$y = x^2 + 2$ and $y = 3$.

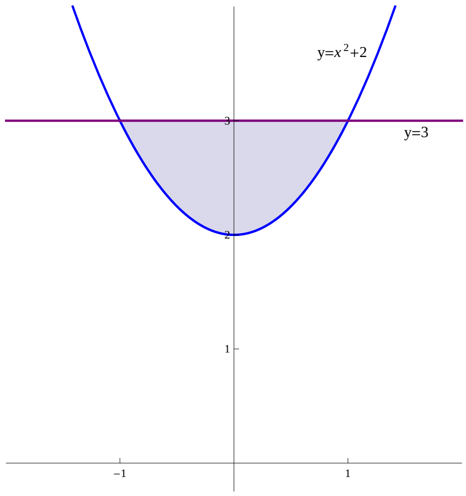
Solution :

$y = x^2 + 2 \implies y - 2 = x^2$ is a parabola opens upwards with vertex $(0, 2)$.

$y = 3$ is a straight line parallel to the x-axis and passes through $(0, 3)$.

Points of intersection of $y = x^2 + 2$ and $y = 3$:

$x^2 + 2 = 3 \implies x^2 = 1 \implies x = \pm 1$.



$$\text{Area} = \int_{-1}^{1} \left[3 - (x^2 + 2)\right] \, dx = \int_{-1}^{1} (1 - x^2) \, dx$$

$$= \left[x - \frac{x^3}{3}\right]_{-1}^{1} = \left(1 - \frac{1}{3}\right) - \left(-1 - \frac{-1}{3}\right) = \frac{2}{3} - \left(-\frac{2}{3}\right) = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

Q.2 Compute the integrals :

(a) $\displaystyle\int \frac{2x + 5}{(x + 2)(x + 1)} \, dx$

(b) $\displaystyle\int \frac{2x + 3}{x^2 + 3x + 6} \, dx$

(c) $\displaystyle\int \frac{3}{x^2 + 2x + 5} \, dx$

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(d) $\int 3x^2 \sin(x^3+3)\ dx$

(e) $\int x \cos x\ dx$

(f) $\int x(x^2+1)^5\ dx$

Solution :

(a) $\int \frac{2x+5}{(x+2)(x+1)}\ dx$

Using the method of partial fractions:

$$\frac{2x+5}{(x+2)(x+1)} = \frac{A_1}{x+2} + \frac{A_2}{x+1}$$

$2x+5 = A_1(x+1) + A_2(x+2)$

Put $x=-2:\ 2(-2)+5 = A_1(-2+1) \implies 1 = -A_1 \implies A_1 = -1$

Put $x=-1:\ 2(-1)+5 = A_2(-1+2) \implies A_2 = -2+5 = 3$

$$\int \frac{2x+5}{(x+2)(x+1)}\ dx = \int \left(\frac{-1}{x+2} + \frac{3}{x+1}\right)\ dx$$

$$= -\int \frac{1}{x+2}\ dx + 3\int \frac{1}{x+1}\ dx = -\ln|x+2| + 3\ln|x+1| + c$$

(b) $\int \frac{2x+3}{x^2+3x+6}\ dx$

Using the formula $\int \frac{f'(x)}{f(x)}\ dx = \ln|f(x)| + c$

$$\int \frac{2x+3}{x^2+3x+6}\ dx = \ln|x^2+3x+6| + c$$

(c) $\int \frac{3}{x^2+2x+5}\ dx$

Using the formula $\int \frac{f'(x)}{a^2+[f(x)]^2}\ dx = \frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right) + c$, where $a > 0$

$$\int \frac{3}{x^2+2x+5}\ dx = 3\int \frac{1}{(x^2+2x+1)+4}\ dx = 3\int \frac{1}{(x+1)^2+2^2}\ dx$$

$$= 3 \times \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + c = \frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + c$$

(d) $\int 3x^2 \sin(x^3+3)\ dx$

Using the formula $\int \sin(f(x))\ f'(x)\ dx = -\cos(f(x)) + c$

$$\int 3x^2 \sin(x^3+3)\ dx = \int \sin(x^3+3)\ (3x^2)\ dx = -\cos(x^3+3) + c$$

(e) $\int x \cos x\ dx$

Using integration by parts :

$u = x \qquad dv = \cos x\ dx$
$du = dx \qquad v = \sin x$

$$\int x\cos x\ dx = x\sin x - \int \sin x\ dx$$

$= x\sin x - (-\cos x) + c = x\sin x + \cos x + c$

(f) $\int x(x^2+1)^5\ dx$

Using the formula $\int [f(x)]^n f'(x)\ dx = \dfrac{[f(x)]^{n+1}}{n+1} + c$, where $n \neq -1$

$$\int x(x^2+1)^5\ dx = \frac{1}{2}\int (x^2+1)^5\ (2x)\ dx = \frac{1}{2}\ \frac{(x^2+1)^6}{6} + c$$

Q.3 Find the area of the srface bounded by the curves :

$y = \sqrt{x}$, $y = 0$ and $y = -x + 2$

Solution :

$y = \sqrt{x} \implies y^2 = x$ is the upper half of a parabola opens to the right with vertex $(0, 0)$.

$y = -x + 2$ is a straight line passes through $(0, 2)$ with slope -1 .

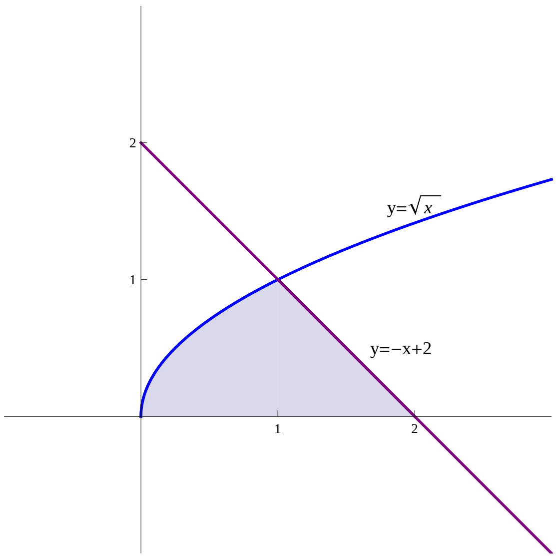
$y = 0$ is the x-axis .

$y = \sqrt{x} \implies y^2 = x$ and $y = -x + 2 \implies x = -y + 2$

Points of intersection of $x = y^2$ and $x = -y + 2$:

$y^2 = -y + 2 \implies y^2 + y - 2 = 0 \implies (y+2)(y-1) = 0$

$\implies y = 1\ ,\ y = -2$



$$\text{Area} = \int_0^1 [(-y+2) - y^2]\ dy = \int_0^1 (-y^2 - y + 2)\ dy = \left[-\frac{y^3}{3} - \frac{y^2}{2} + 2y\right]_0^1$$

$$= \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - (0 - 0 + 0) = \frac{-2 - 3 + 12}{6} = \frac{7}{6}$$

Another solution :

$$\text{Area} = \int_0^1 \sqrt{x}\ dx + \int_1^2 (-x+2)\ dx = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + \left[-\frac{x^2}{2} + 2x\right]_1^2$$

$$= \left[\frac{2}{3} - 0\right] + \left[(-2+4) - \left(-\frac{1}{2} + 2\right)\right] = \frac{2}{3} + 2 - \frac{3}{2} = \frac{4 + 12 - 9}{6} = \frac{7}{6}$$

M 104 - GENERAL MATHEMATICS -2-

Dr. Tariq A. AlFadhel[6]

Solution of the Final Exam

Second semester 1435-1436 H

Q.1 **(a)** Compute $\mathbf{BA}+\mathbf{AB}$ for $\mathbf{A}=\begin{pmatrix}2&2&0\\1&0&1\\0&1&3\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}1&2&1\\2&1&2\\1&0&1\end{pmatrix}$

(b) Compute the determinant $\begin{vmatrix}3&1&-2\\1&2&3\\4&-1&0\end{vmatrix}$.

(c) Solve by Gauss Elimination Method : $\begin{cases} x+y+z=3\\ x-y-z=1\\ 2x+y-z=3\end{cases}$

Solution :

(a) $\mathbf{BA}=\begin{pmatrix}1&2&1\\2&1&2\\1&0&1\end{pmatrix}\begin{pmatrix}2&2&0\\1&0&1\\0&1&3\end{pmatrix}$

$$=\begin{pmatrix}2+2+0&2+0+1&0+2+3\\4+1+0&4+0+2&0+1+6\\2+0+0&2+0+1&0+0+3\end{pmatrix}=\begin{pmatrix}4&3&5\\5&6&7\\2&3&3\end{pmatrix}$$

$\mathbf{AB}=\begin{pmatrix}2&2&0\\1&0&1\\0&1&3\end{pmatrix}\begin{pmatrix}1&2&1\\2&1&2\\1&0&1\end{pmatrix}$

$$=\begin{pmatrix}2+4+0&4+2+0&2+4+0\\1+0+1&2+0+0&1+0+1\\0+2+3&0+1+0&0+2+3\end{pmatrix}=\begin{pmatrix}6&6&6\\2&2&2\\5&1&5\end{pmatrix}$$

$$\mathbf{BA}+\mathbf{AB}=\begin{pmatrix}4&3&5\\5&6&7\\2&3&3\end{pmatrix}+\begin{pmatrix}6&6&6\\2&2&2\\5&1&5\end{pmatrix}=\begin{pmatrix}10&9&11\\7&8&9\\7&4&8\end{pmatrix}$$

(b) Using Sarrus Method

$$\begin{matrix}3&1&-2&3&1\\1&2&3&1&2\\4&-1&0&4&-1\end{matrix}$$

$$\begin{vmatrix}3&1&-2\\1&2&3\\4&-1&0\end{vmatrix}=(0+12+2)-(-16-9+0)=14-(-25)=14+25=39$$

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(c) Using Gauss Elimination Method

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -1 & -1 & 1 \\ 2 & 1 & -1 & 3 \end{array}\right) \xrightarrow{-R_1+R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -2 & -2 \\ 2 & 1 & -1 & 3 \end{array}\right)$$

$$\xrightarrow{-2R_1+R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -2 & -2 \\ 0 & -1 & -3 & -3 \end{array}\right) \xrightarrow{-\frac{1}{2}R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & -3 & -3 \end{array}\right)$$

$$\xrightarrow{R_2+R_3} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 \end{array}\right)$$

$-2z = -2 \implies z = 1$

$y + z = 1 \implies y + 1 = 1 \implies y = 0$

$x + y + z = 3 \implies x + 0 + 1 = 3 \implies x = 2$

The solution of the linear system is $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

Q.2 Find all the elements of the two following conic sections and sketch their graphs :

(a) $9y^2 + 4x^2 + 18y - 8x = 23$

(b) $9x^2 - 4y^2 - 16y - 18x - 43 = 0$

Solution :

(a) $9y^2 + 4x^2 + 18y - 8x = 23$

$4x^2 - 8x + 9y^2 + 18y = 23$

$4(x^2 - 2x) + 9(y^2 + 2y) = 23$

By completing the square

$4(x^2 - 2x + 1) + 9(y^2 + 2y + 1) = 23 + 4 + 9$

$4(x-1)^2 + 9(y+1)^2 = 36$

$$\frac{4(x-1)^2}{36} + \frac{9(y+1)^2}{36} = 1$$

$$\frac{(x-1)^2}{9} + \frac{(y+1)^2}{4} = 1$$

The conic sectionis an ellipse with center $P = (1, -1)$

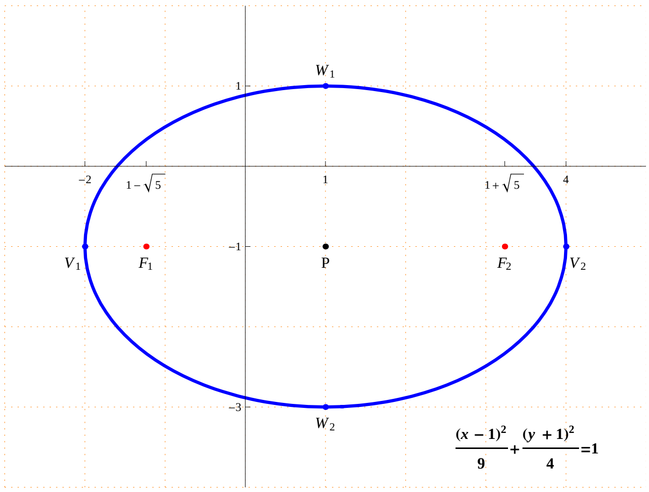
$a^2 = 9 \implies a = 3$ and $b^2 = 4 \implies b = 2$

$c^2 = a^2 - b^2 = 9 - 4 = 5 \implies c = \sqrt{5}$

The vertices are $V_1 = (-2, -1)$ and $V_2 = (4, -1)$

The Foci are $F_1 = \left(1 - \sqrt{5}, -1\right)$ and $F_2 = \left(1 + \sqrt{5}, -1\right)$

The end-points of the minor axis are $W_1 = (1, 1)$ and $W_2 = (1, -3)$



(b) $9x^2 - 4y^2 - 16y - 18x - 43 = 0$

$9x^2 - 18x - 4y^2 - 16y = 43$

$9(x^2 - 2x) - 4(y^2 + 4y) = 43$

By completing the square

$9(x^2 - 2x + 1) - 4(y^2 + 4y + 4) = 43 + 9 - 16$

$9(x - 1)^2 - 4(y + 2)^2 = 36$

$$\frac{9(x-1)^2}{36} - \frac{4(y+2)^2}{36} = 1$$

$$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{9} = 1$$

The conic section is a hyperbola with transverse axis parallel to the x-axis.

The ceneter is $P = (1, -2)$.

$a^2 = 4 \implies a = 2$ and $b^2 = 9 \implies b = 3$.

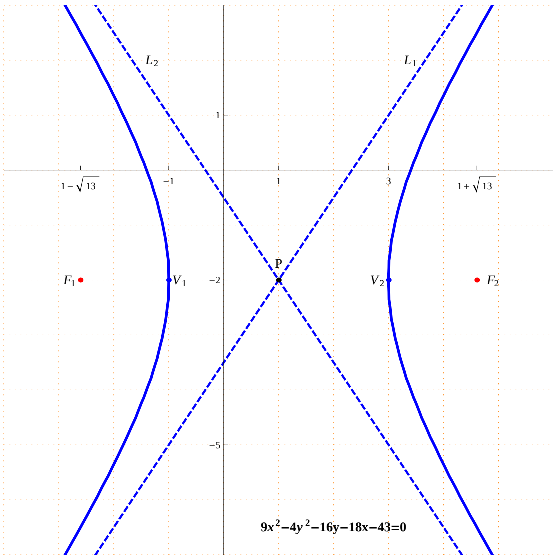
$c^2 = a^2 + b^2 = 9 + 4 = 13 \implies c = \sqrt{13}$.

The vertices are $V_1 = (-1, -2)$ and $V_2 = (3, -2)$.

The foci are $F_1 = \left(1 - \sqrt{13}, -2\right)$ and $F_2 = \left(1 + \sqrt{13}, -2\right)$.

The equations of the asymptotes are $L_1 \ : \ y + 2 = \dfrac{3}{2}\ (x - 1)$ and

$L_2 \; : \; y + 2 = -\dfrac{3}{2}\,(x-1)$



Q.3 **(a)** Compute the integrals :

(i) $\displaystyle\int \frac{2}{(x-1)(x+1)}\,dx$ (ii) $\displaystyle\int x\sin x\; dx$ (iii) $\displaystyle\int \frac{2x-4}{x^2-4x+5}\,dx$

(b) Find the area of the region delimited by the curves :

$y = 2$ and $y = x^2 + 1$.

(c) The region R in the first quadrant lying between the curves $y = 0$ and $y = 1 - x^2$ is rotated about the x-axis to form a solid of revolution S . Find the volume of S.

Solution :

(a) (i) $\displaystyle\int \frac{2}{(x-1)(x+1)}\,dx$

Using the method of partial fractions

$$\frac{2}{(x-1)(x+1)} = \frac{A_1}{x-1} + \frac{A_2}{x+1}$$

$2 = A_1(x+1) + A_2(x-1)$

Put $x = 1$: $2 = 2A_1 \implies A_1 = 1$

Put $x = -1$: $2 = -2A_2 \implies A_2 = -1$

$$\frac{2}{(x-1)(x+1)} = \frac{1}{x-1} + \frac{-1}{x+1}$$

$$\int \frac{2}{(x-1)(x+1)}\ dx = \int \left(\frac{1}{x-1} + \frac{-1}{x+1}\right)\ dx$$

$$= \int \frac{1}{x-1}\ dx - \int \frac{1}{x+1}\ dx = \ln|x-1| - \ln|x+1| + c$$

(ii) $\int x \sin x\ dx$

Using integration by parts

$u = x \qquad dv = \sin x\ dx$

$du = dx \qquad v = -\cos x$

$$\int x\sin x\ dx = -x\cos x - \int -\cos x\ dx = -x\cos x + \int \cos x\ dx$$

$= -x\cos x + \sin x + c$

(iii) $\int \frac{2x-4}{x^2-4x+5}\ dx$

Using the formula $\int \frac{f'(x)}{f(x)}\ dx = \ln|f(x)| + c$

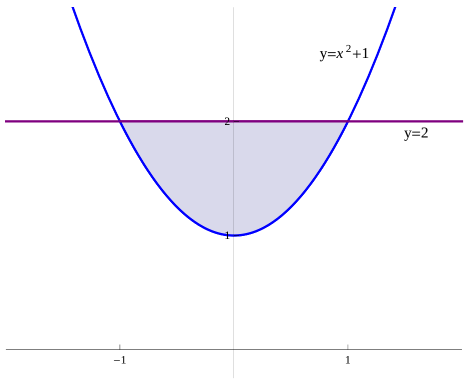
$$\int \frac{2x-4}{x^2-4x+5}\ dx = \ln|x^2-4x+5| + c$$

(b) $y = 2$ is a straight line parallel to the x-axis and passes through $(0, 2)$

$y = x^2 + 1 \implies y - 1 = x^2$ is a parabola with vertex $(0, 1)$ and opens upwards

Points of intersection of $y = 2$ and $y = x^2 + 1$:

$x^2 + 1 = 2 \implies x^2 - 1 = 0 \implies (x-1)(x+1) = 0 \implies x = -1\ ,\ x = 1$



$$\text{Area} = \int_{-1}^{1} \left[2 - (x^2+1)\right]\ dx = \int_{-1}^{1} (1 - x^2)\ dx = \left[x - \frac{x^3}{3}\right]_{-1}^{1}$$

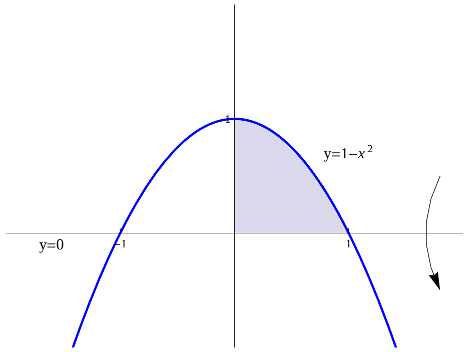
$$= \left(1 - \frac{1}{3}\right) - \left(-1 - \frac{-1}{3}\right) = \frac{2}{3} - \left(-1 + \frac{1}{3}\right) = \frac{2}{3} - \frac{-2}{3} = \frac{4}{3}$$

(c) $y = 0$ is the x-axis.

$y = 1 - x^2 \implies (y-1) = -x^2$ is a parabola opens downwards with vertex $(0, 1)$.

Points of intersection of $y = 1 - x^2$ and $y = 2$:

$1 - x^2 = 2 \;\Rightarrow\; x^2 - 1 = 0 \;\Rightarrow\; (x-1)(x+1) = 0 \;\Rightarrow\; x = -1\ ,\ x = 1$



Using Disk Method :

$$\text{Volume} = \pi \int_0^1 (1-x^2)^2\ dx = \pi \int_0^1 (1 - 2x^2 + x^4)\ dx = \left[x - \frac{2x^2}{3} + \frac{x^5}{5}\right]_0^1$$

$$= \pi\left[\left(1 - \frac{2}{3} + \frac{1}{5}\right) - (0 - 0 + 0)\right] = \frac{15 - 10 + 3}{15}\pi = \frac{8}{15}\pi$$

Q.4 **(a)** Find f_x , f_y and f_z for the function $f(x, y, z) = x^2y^3 + z^5ye^y$

(b) Solve the differential equation $xdy + y^2dx = 0$, $y(1) = 1$

Solution :

(a) $f_x = 2xy^3 + 0 = 2xy^3$

$f_y = x^2(3y^2) + z^5(1 \times e^y + ye^y) = 3x^2y^2 + z^5(e^y + ye^y)$

$f_z = 0 + 5z^4ye^y = 5z^4ye^y$

(b) $xdy + y^2dx = 0$

$x\ dy = -y^2\ dx$

$$\frac{-1}{y^2}\ dy = \frac{1}{x}\ dx$$

$$-y^{-2}\ dy = \frac{1}{x}\ dx$$

It is a separable differential equation

$$\int -y^2 \ dy = \int \frac{1}{x} \ dx$$

$$-\frac{y^{-1}}{-1} = \ln|x| + c$$

$$y^{-1} = \ln|x| + c$$

The general solution of the differential equation is :

$$y = \frac{1}{\ln|x| + c}$$

$$y(1) = 1 \implies 1 = \frac{1}{\ln(1) + c} = \frac{1}{0 + c} = \frac{1}{c} \implies c = 1$$

The particular solution of the differential equation is :

$$y = \frac{1}{\ln|x| + 1}$$