Department of Mathematics

M-106

First Mid Term Exam
Second Semester(1428/1429)

Max. Marks: 20 **Time: 90 minutes**

Name:………………………….Number:…………..

Marks:

Multiple Choice(1-to-10)…………[ ]

Question (11)…………[ ]

Question (12)…………[ ]

Question (13)…………[ ]

Question (14)………...[ ]

Total……………………………….[ ]

Marking Scheme:[From Q.No.1-to-Q.No.10 one mark each]

Multiple Choices

Q.No.	1	2	3	4	5	6	7	8	9	10
{a,b,c,d}	a	b	b	a	a	a	b	b	b	b

Q.No:1 The sum $\sum_{k=1}^{10} k(k-2)^2$ is equal to:

(a) 1705 (b) 1155 (c) 1255 (d) None of these.

Q.No:2 If $f(x) = 4x^3$, then the number z that satisfies the conclusion of the Mean Values theorem on [0,1] is

(a) 1 (b) $\left(\frac{1}{4}\right)^{1/3}$ (c) $-\left(\frac{1}{4}\right)^{1/3}$ (d) None of these.

Q.No:3 The average value of the function $f(x) = 2x - x^2$ on the interval [0,1] is:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) None of these.

Q.No:4 If $f(x) = \int_x^{x^2} Sin(t^2)dt$ then $f'(x)$ is equal to:

(a) $2xSin(x^4) - Sin(x^2)$, (b) $2xSin(x^4) + Sin(x^2)$,
(c) $2xCos(x^4) - Cos(x^2)$, (d) None of these.

Q .No:5 The value of the integral $\int_1^2 \frac{e^{3/x}}{x^2} dx$ is equal to :

(a) $e^{3/2} + e^3$, (b) $-\frac{1}{3}(e^{3/2} - e^3)$, (c) $\frac{1}{3}(e^3 - e^{3/2})$ (d) None of these.

Q.No:6 If $y = (x+1)^{Cos(x)}$, then y' at $x = 0$ is equal to

(a) 1, (b) -1, (c) 2, (d) None of these.

Q.No:7 The value of the integral $\int_0^1 \frac{3x^2 + 2x}{x^3 + x^2 + 2} dx$ is equal to

(a) $\ln(4)$, (b) $\ln(2)$, (c) $2\ln(4)$, (d) None of these.

Q.No:8 If $\log_4(x^2) = 1$, then x is equal to:

(a) 1 or -1, (b) 2 or -2, (c) 1 , 2, (d) None of these.

Q.No:9 The value of the integral $\int \frac{Cosx}{1 + Sin^2 x} dx$ is equal to

(a) $\frac{1}{2}\ln(1 + Sin^2 x)$, (b) $\tan^{-1}(Sin(x)) + c$, (c) $\tanh^{-1}(Sin(x)) + c$ (d) None of these.

Q.No:10 The value of the integral $\int \frac{Cosx}{1 - Sin^2 x} dx$ is equal to

(a) $-\frac{1}{2}\ln(1 - Sin^2 x)$, (b) $\tanh^{-1}(Sin(x)) + c$, (c) $\tan^{-1}(Sin(x)) + c$ (d) None of these.

Question No: 11 Use the Simpson's rule to approximate the integral

$$\int_{-\pi/3}^{\pi/3} \tan(x)dx \text{ with } n = 4.$$

Solution: $\Delta x = \dfrac{\pi/3-(-\pi/3)}{4} = \dfrac{\pi}{6}$. $x_0 = -\dfrac{\pi}{3}, x_1 = -\dfrac{\pi}{6}, x_2 = 0, x_3 = \dfrac{\pi}{6}, x_4 = \dfrac{\pi}{3}$

By Simpson's rule we have $\displaystyle\int_{-\pi/3}^{\pi/3} \tan(x)dx \approx \frac{\pi/3-(-\pi/3)}{3(4)}$

$$\left\{\tan(-\frac{\pi}{3}) + 4\tan(-\frac{\pi}{6}) + 2\tan(0) + 4\tan(\frac{\pi}{6}) + \tan(\frac{\pi}{6})\right\}$$

$$= \frac{2\pi/3}{12}\{-1.7 - 4(.57) + 2(0) + 4(.57) + (1.7)\} = 0.$$

Question No: 12 Evaluate the integral $\displaystyle\int_0^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^3}dx$

Solution: Put $u = \sqrt{x}+1$, $du = \dfrac{1}{2\sqrt{x}}dx$, $2du = \dfrac{1}{\sqrt{x}}dx$.

$$\int_0^4 \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^3}dx = \int_1^3 \frac{1}{u^3}(2du) = 2\left[-\frac{1}{2u^2}\right]_1^3 = 2\left[-\frac{1}{18}+\frac{1}{2}\right] = \frac{8}{9}$$

Question No: 13 Find y' if $y^2 + \ln\left(\dfrac{x}{y}\right) - 4x = -3$.

Solution: $y^2 + \ln(x) - \ln(y) - 4x = -3$.

Now on differentiating with respect to x we get:

$$2yy' + \frac{1}{x} - \frac{1}{y}y' - 4 = 0 \ . \quad \left(2y - \frac{1}{y}\right)y' = 4 - \frac{1}{x}.$$

$$y' = \frac{\left(4 - 1/x\right)}{\left(2y - 1/y\right)}.$$

Question No: 14 Evaluate the integral $\int \frac{1}{x\sqrt{x^8-16}}dx$.

Solution: $\int \frac{1}{x\sqrt{x^8-16}}dx = \int \frac{1}{x\sqrt{(x^4)^2-(4)^2}}dx$

Now put $u = x^4 \Rightarrow du = 4x^3dx$, $\frac{1}{4}du = x^3dx$

$$\int \frac{1}{x\sqrt{(x^4)^2-(4)^2}}dx = \int \frac{x^3}{x^4\sqrt{(x^4)^2-(4)^2}}dx \text{ (multiplied and divide by } x^3 \text{)}$$

$$= \int \frac{1}{u\sqrt{(u)^2-(4)^2}}\frac{1}{4}du = \frac{1}{4}\int \frac{1}{u\sqrt{(u)^2-(4)^2}}du$$

$$= \frac{1}{4}\left[\frac{1}{4}Sec^{-1}\left(\frac{u}{4}\right)\right] + c = \frac{1}{16}Sec^{-1}(x^4) + c$$