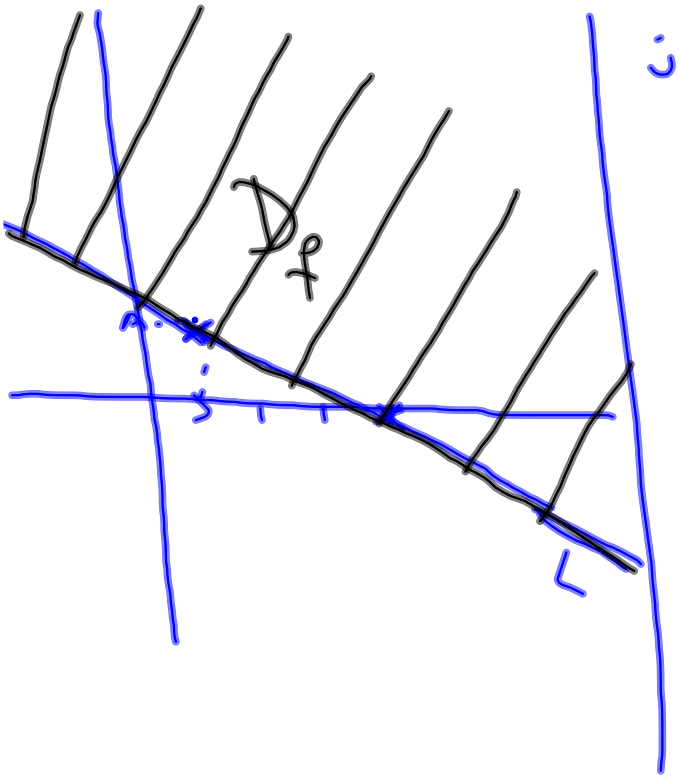
الفصل الأول 37/36

س ١ (أ)

$$f(x,y)=\sqrt{x+3y-4}$$

$f(x,y)$ معرفة إذا وفقط إذا كان :

$$x+3y-4\geq 0$$



$$L: x+3y-4=0$$

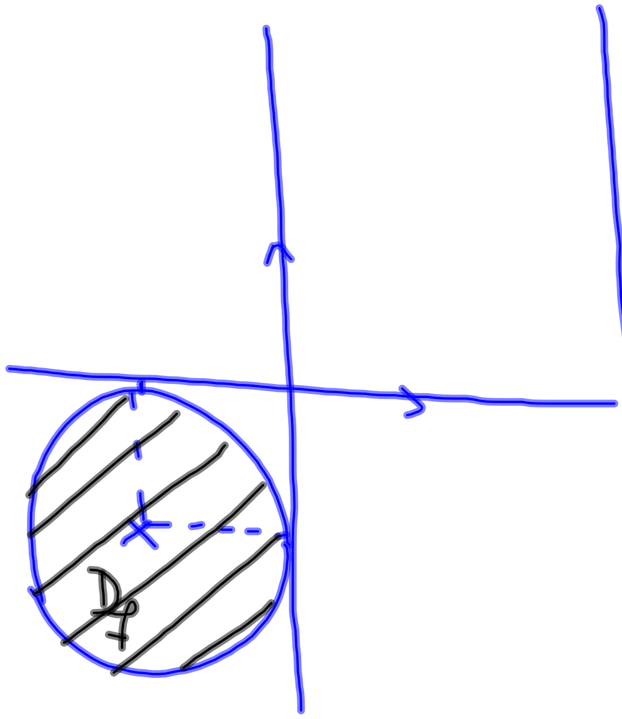
x	1	4
y	1	0

النقطة $(0,0)$

$$0+3(0)-4 \ngeq 0$$

(ب)

$$f(x,y)=\ln\left(1-(x+1)^2-(y+1)^2\right)$$



$f(x,y)$ معرفة إذا وفقط إذا كان

$$1-(x+1)^2-(y+1)^2>0$$

$$\Leftrightarrow \quad 1^2>(x+1)^2+(y+1)^2$$

$$\Leftrightarrow \quad \left((x+1)^2+(y+1)^2<1^2\right)$$

إذن D_f هو القرص المفتوح

مركزه $(-1,-1)$ ونصف قطره $r=1$

(2) (1) نهاية: $\dfrac{xy^2}{x^2+y^2}$ عند ما تقترب (x,y) من $(0,0)$

لدينا: $0 \le \dfrac{y^2}{x^2+y^2} \le 1$ لكل $(x,y) \ne (0,0)$

فإن: $0 \le |x| \dfrac{y^2}{x^2+y} \le |x|$

$0 \le |f(x,y)| = \left|\dfrac{xy^2}{x^2+y^2}\right| \le |x|$

وبما أن $\lim\limits_{(x,y)\to(0,0)} |x| = 0$

فإن $\lim\limits_{(0,0)} |f(x,y)| = 0$

فإن $\lim\limits_{(0,0)} f(x,y) = 0$

(ب)

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{x y^2}{x^4+y^4} = \lim_{x\to 0} \frac{x\, x^2}{x^4+x^4}$$

$$= \lim_{x\to 0} \frac{x^3}{2x^4} = \lim_{x\to 0} \frac{1}{2x} = \pm\infty$$

إذن $\lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{xy^2}{x^4+y^4}$ غير موجودة

س 2 :

$$f(x,y)=\begin{cases}\dfrac{xy^2}{x^4+y^4}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

(1) $D_f=\mathbb{R}^2$

(2) لدينا $(x,y)\xmapsto{g}\dfrac{xy^2}{x^4+y^4}$ دالة كسرية متصلة عند كل نقطة من مجالها $\mathbb{R}^2\setminus\{(0,0)\}$

عند النقطة $(0,0)$ نعلم أن $\lim\limits_{(0,0)}\dfrac{xy^2}{x^4+y^4}$ غير موجودة.

فإن f ليست متصلة عند النقطة $(0,0)$.

وبالتالي f متصلة عند كل نقطة من $\mathbb{R}^2\setminus\{(0,0)\}$

(3)

$$\lim_{x\to 0} \frac{f(x,0)-f(0,0)}{x-0} = \lim_{x\to 0} \frac{\frac{0}{x^4}-0}{x} = \lim_{x\to 0} 0 = 0$$

نازن

$$\boxed{f_x(0,0) = 0}$$

$$\lim_{y\to 0} \frac{f(0,y)-f(0,0)}{y-0} = \lim_{y\to 0} \frac{\frac{0}{y^4}-0}{y} = \lim_{y\to 0} 0 = 0$$

نازن

$$\boxed{f_y(0,0) = 0}$$

(4) عند النقطة $(x,y) \neq (0,0)$

$$f_x(x,y) = \frac{y^2(x^4+y^4) - xy^2\,4x^3}{(x^4+y^4)^2} = \frac{y^2(-3x^4+y^4)}{(x^4+y^4)^2}$$

$$f_y(x,y) = \frac{2xy(x^4+y^4) - xy^2\,4y^3}{(x^4+y^4)^2} = \frac{2xy(x^4-y^4)}{(x^4+y^4)^2}$$

$$f_y(x,y) = \begin{cases} \dfrac{2xy(x^4-y^4)}{(x^4+y^4)^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

$$f_x(x,y) = \begin{cases} \dfrac{y^2(-3x^4+y^4)}{(x^4+y^4)^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(5)

$\frac{\partial^2 f}{\partial x \partial y}(0,0)$?

$$\lim_{y \to 0} \frac{f_x(0,y) - f_x(0,0)}{y - 0} = \lim_{y \to 0} \frac{\frac{y^6}{y^8} - 0}{y} = \lim_{y \to 0} \frac{1}{y^3} = \pm\infty$$

فإن $\frac{\partial^2 f}{\partial x \partial y}(0,0)$ غير موجود.

$\frac{\partial^2 f}{\partial y \partial x}(0,0)$?

$$\lim_{x \to 0} \frac{f_y(x,0) - f_y(0,0)}{x - 0} = \lim_{x \to 0} \frac{\frac{0}{x^5} - 0}{x} = \lim_{x \to 0} 0 = 0$$

فإن

$$\boxed{\frac{\partial^2 f}{\partial y \partial x}(0,0) = 0}$$

$\frac{\partial^2 w}{\partial u^2}+\frac{\partial^2 w}{\partial v^2}=?$ $\quad y=u-v \quad , x=2u+v \quad , w=f(x,y)$: س 3

$$\frac{\partial w}{\partial u}=\frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial u}+\frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial u}=\frac{\partial w}{\partial x}\cdot 2+\frac{\partial w}{\partial y}\cdot 1=2\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}$$

$$\frac{\partial^2 w}{\partial u^2}=\frac{\partial}{\partial u}\left(\frac{\partial w}{\partial u}\right)=\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial u}\right)\frac{\partial x}{\partial u}+\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial u}\right)\frac{\partial y}{\partial u}$$

$$=\frac{\partial}{\partial x}\left(2\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}\right)\cdot 2+\frac{\partial}{\partial y}\left(2\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}\right)\cdot 1$$

$$=4\frac{\partial^2 w}{\partial x^2}+2\frac{\partial^2 w}{\partial y\partial x}+2\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 w}{\partial y^2}=4\frac{\partial^2 w}{\partial x^2}+4\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 w}{\partial y^2}$$

$\frac{\partial^2 w}{\partial u^2}$

$$\frac{\partial w}{\partial v}=\frac{\partial w}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial v}=\frac{\partial w}{\partial x}\cdot 1+\frac{\partial w}{\partial y}\cdot(-1)=\frac{\partial w}{\partial x}-\frac{\partial w}{\partial y}$$

$$\frac{\partial^2 w}{\partial v^2}=\frac{\partial}{\partial v}\left(\frac{\partial w}{\partial v}\right)=\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial v}\right)\frac{\partial x}{\partial v}+\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial v}\right)\frac{\partial y}{\partial v}$$

$$=\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}-\frac{\partial w}{\partial y}\right)\cdot 1+\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}-\frac{\partial w}{\partial y}\right)(-1)=\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 w}{\partial y\partial x}-\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 w}{\partial y^2}$$

$$\boxed{\frac{\partial^2 w}{\partial v^2}=\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 w}{\partial y^2}}$$

$$\boxed{\frac{\partial^2 w}{\partial u^2}+\frac{\partial^2 w}{\partial v^2}=5\frac{\partial^2 w}{\partial x^2}+2\frac{\partial^2 w}{\partial x\partial y}+2\frac{\partial^2 w}{\partial y^2}}$$

وبالتالي