# Lexical Analysis

Implementation: Finite Automata

# Outline

- Specifying lexical structure using regular expressions
- Finite automata
  - Deterministic Finite Automata (DFAs)
  - Non-deterministic Finite Automata (NFAs)
- Implementation of regular expressions

    RegExp => NFA => DFA => Tables

# Notation

- There is variation in regular expression notation

- Union: $A \mid B \equiv A + B$
- Option: $A + \varepsilon \equiv A?$
- Range: ‘a’ +’ b’ +…+’ z’ $\equiv$ [a-z]
- Excluded range:

  complement of [a-z] $\equiv$ [^a-z]

# Regular Expressions in Lexical Specification

- Given a string s and a reg. exp. R, is

  $s \in L(R)$ ?

- But a yes/no answer is not enough!

- Instead: partition the input into tokens

- We adapt regular expressions to this goal

# Regular Expressions => Lexical Spec.

1. Write a rexp for the lexemes of each token
   - Number = digit +
   - Keyword = ‘if’+ ‘else’+ …
   - Identifier = letter (letter + digit)*
   - OpenPar = ‘(‘
   - …

# Regular Expressions => Lexical Spec.

2. Construct R, matching all lexemes for all tokens

   R = Keyword + Identifier + Number + …

   $= R_1 + R_2 + \ldots$

# Regular Expressions => Lexical Spec.

3. Let input be $x_1 \ldots x_n$

   For $1 \leq i \leq n$ check

   $x_1 \ldots x_i \in L(R)$

4. If success, then we know that

   $x_1 \ldots x_i \in L(R_j)$ for some $j$

5. Remove $x_1 \ldots x_i$ from input and go to (3)

# Ambiguities (1)

- There are ambiguities in the algorithm
- How much input is used? What if
  - $x_1 \ldots x_i \in L(R)$ and also
  - $x_1 \ldots x_K \in L(R)$
- e.g. = and ==
- Rule: Pick longest possible string in $L(R)$
  - The “maximal munch”
  - We as humans do that.

# Ambiguities (2)

- Which token is used? What if
  - $x_1 \ldots x_i \in L(R_j)$ and also
  - $x_1 \ldots x_i \in L(R_k)$

- e.g. 'if' could be an identifier or a keyword;
- which one to choose?
- Rule: use rule listed first ($j$ if $j < k$)
  - Treats "if" as a keyword, not an identifier
- i.e. the one listed first is given higher priority

# Error Handling

- What if
  - No rule matches a prefix of input ?
- Problem: Can’t just get stuck …
- A compiler needs to give feedback to the user e.g. where the error is in the file (line number)
- Solution:
  - Write a rule matching all “bad”strings
  - Put it last (lowest priority)

# Summary

- Regular expressions provide a concise notation for string patterns
- Use in lexical analysis requires small extensions
  - To resolve ambiguities
  - To handle errors
- Good algorithms known
  - Require only single pass over the input
  - Few operations per character (table lookup)

# Finite Automata

- Regular expressions = specification
- Finite automata = implementation
- A finite automaton consists of
  - An input alphabet $\Sigma$
  - A set of states $S$
  - A start state $n$
  - A set of accepting states $F \subseteq S$
  - A set of transitions $\text{state} \rightarrow^{\text{input}} \text{state}$

# Finite Automata

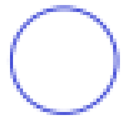
- Transition

$$s_1 \rightarrow^{a} s_2$$

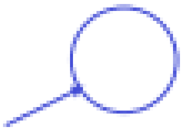
- Is read
- In state $s_1$ on input "a" go to state $s_2$
- If end of input and in accepting state => accept
- Otherwise => reject
  - If it terminates in state s that not a member of F
  - Or it gets stuck because there is not transition from state s1 on input a (i.e. never reaches the end of input).

# Finite Automata State Graphs

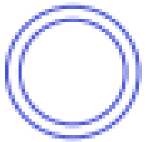
- *A state*



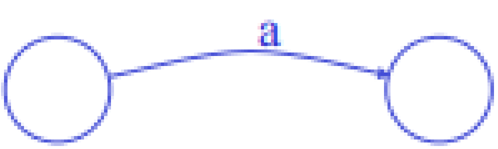
- The start state



- *An accepting state*

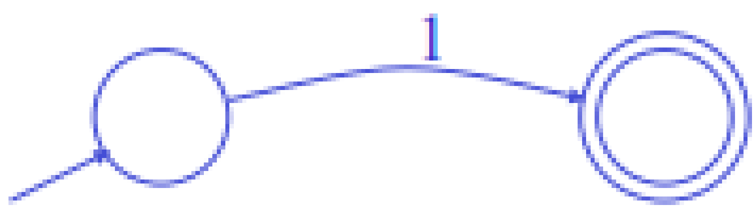


- *A transition*



# A Simple Example

- A finite automaton that accepts only "1"

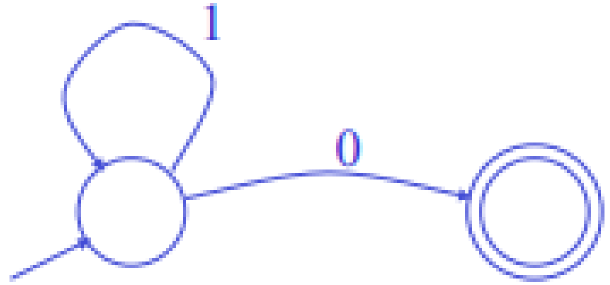


# Another Simple Example

- A finite automaton accepting any number of

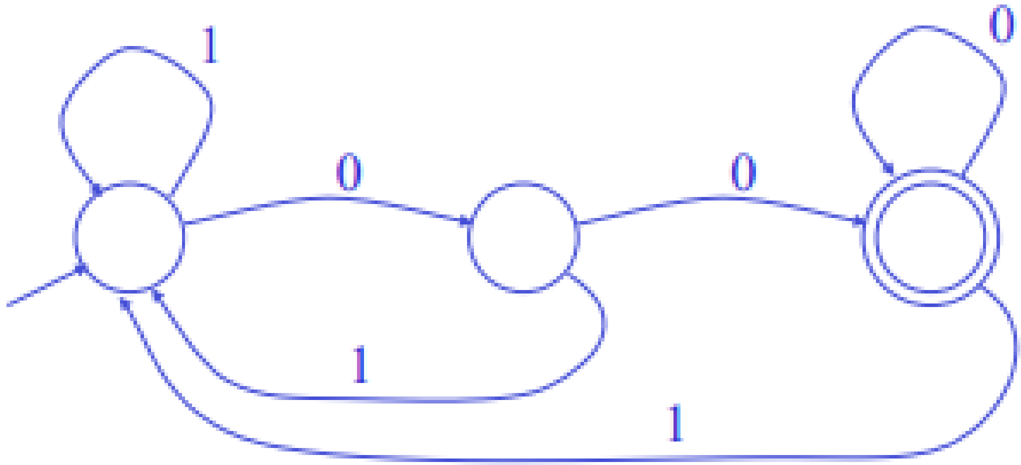
1's followed by a single 0

- Alphabet: {0,1}



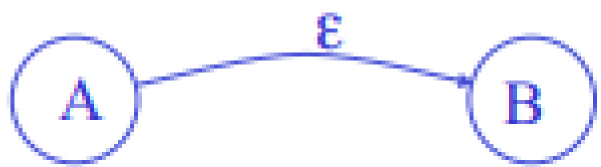
# And Another Example

- Alphabet {0,1}
- What language does this recognize?



# Epsilon Moves

- Another kind of transition: ε-moves




- Machine can move from state A to state B without reading input

# Deterministic and Nondeterministic Automata

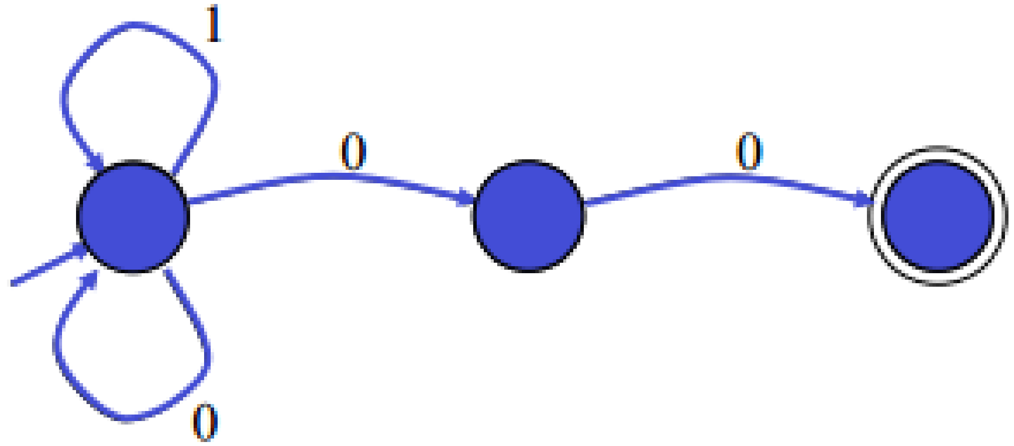
- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves

# Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

# Acceptance of NFAs

- An NFA can get into multiple states




- Input: 1 0 0
- States: {A} {A,B} {A,B,C}
- Rule: NFA accepts if it can get to a final state

# NFA vs. DFA

- NFAs and DFAs recognize the same set of languages (regular languages)

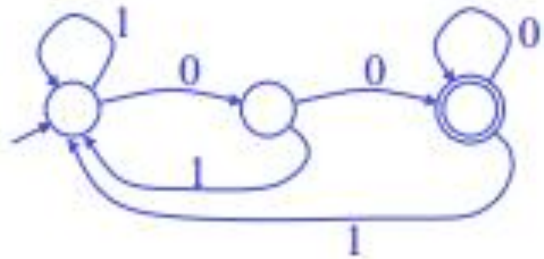
- DFAs are faster to execute
  - There are no choices to consider

# NFA vs. DFA

- For a given language NFA can be simpler than DFA

NFA

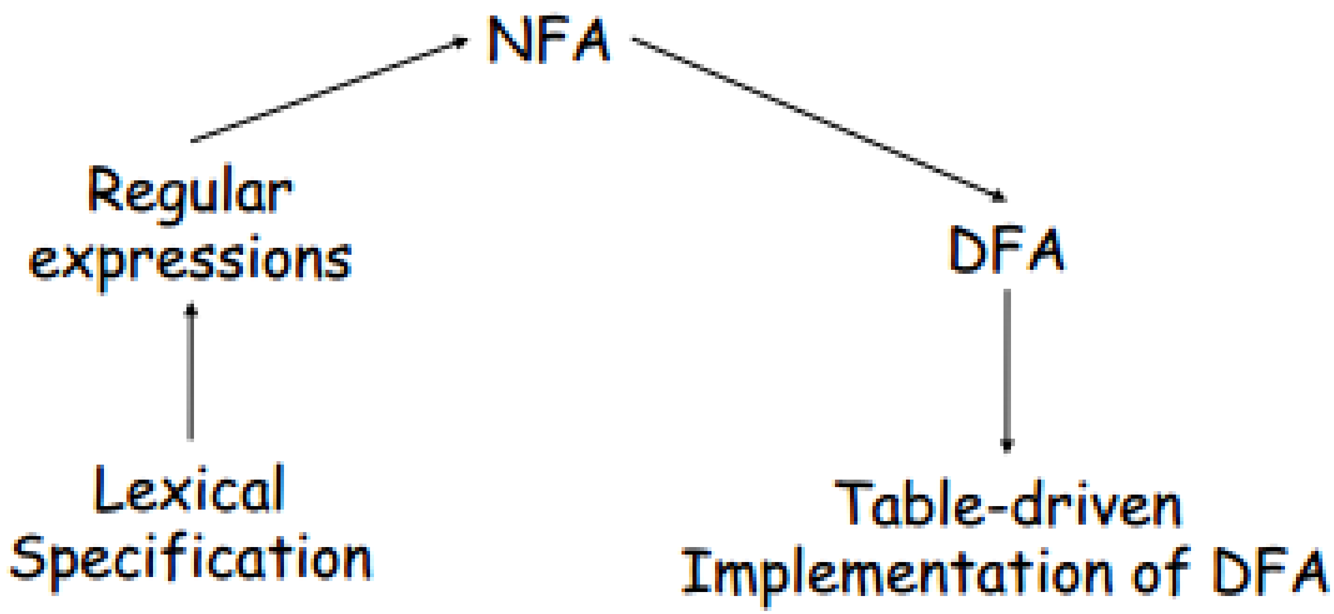

DFA




- DFA can be exponentially larger than NFA

# Regular Expressions to Finite Automata

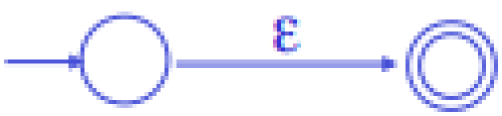
- High-level sketch



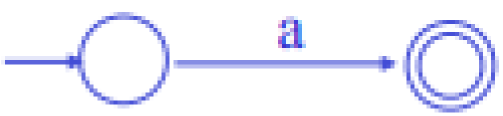
# Regular Expressions to NFA (1)

- For each kind of rexp, define an NFA
  - Notation: NFA for rexp *M*


- For ε

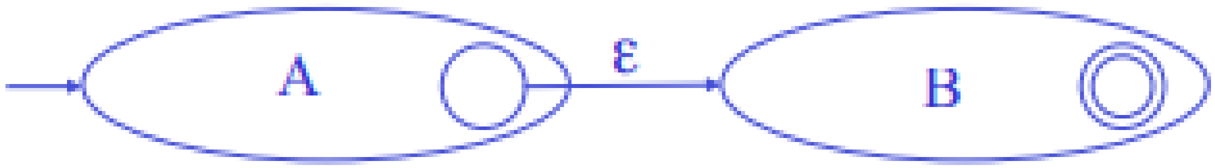



- For input *a*

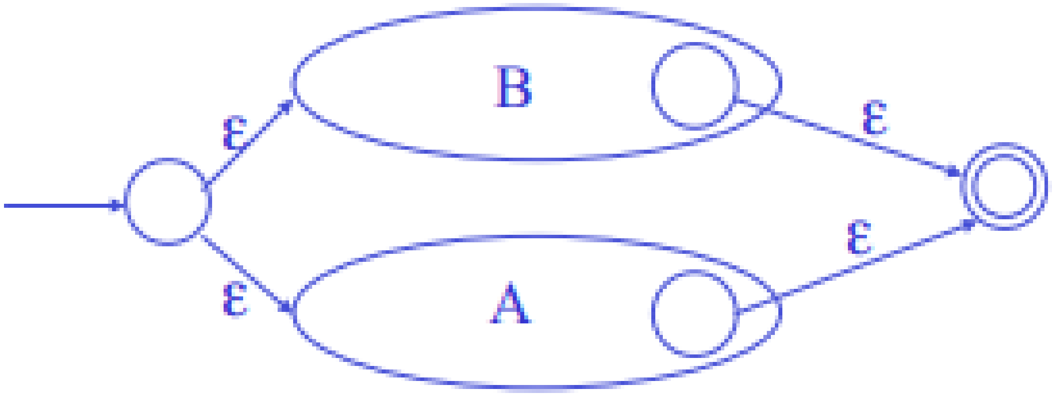


# Regular Expressions to NFA (2)

- For *AB*

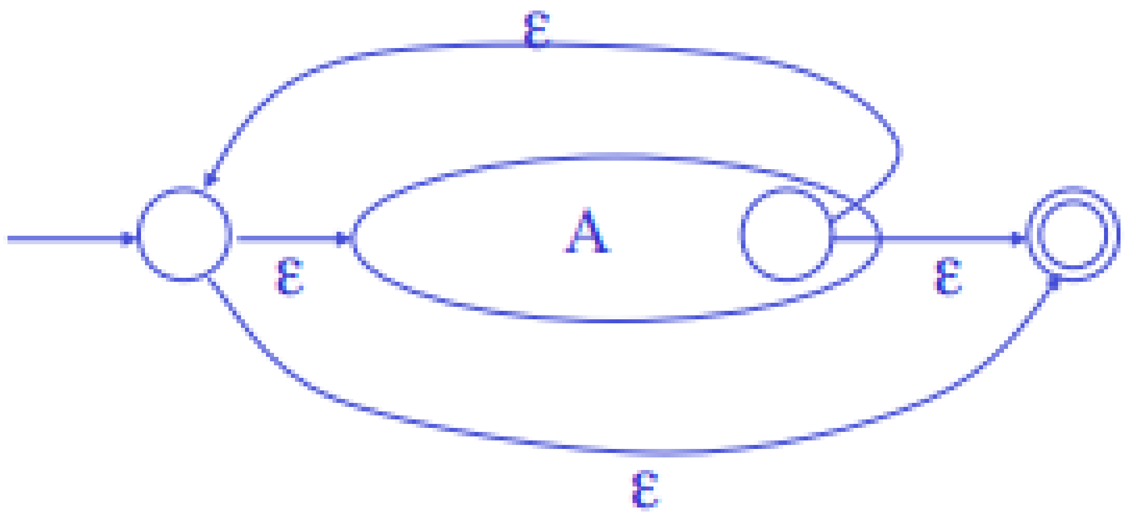



- For *A* + B



# Regular Expressions to NFA (3)

- For *A**

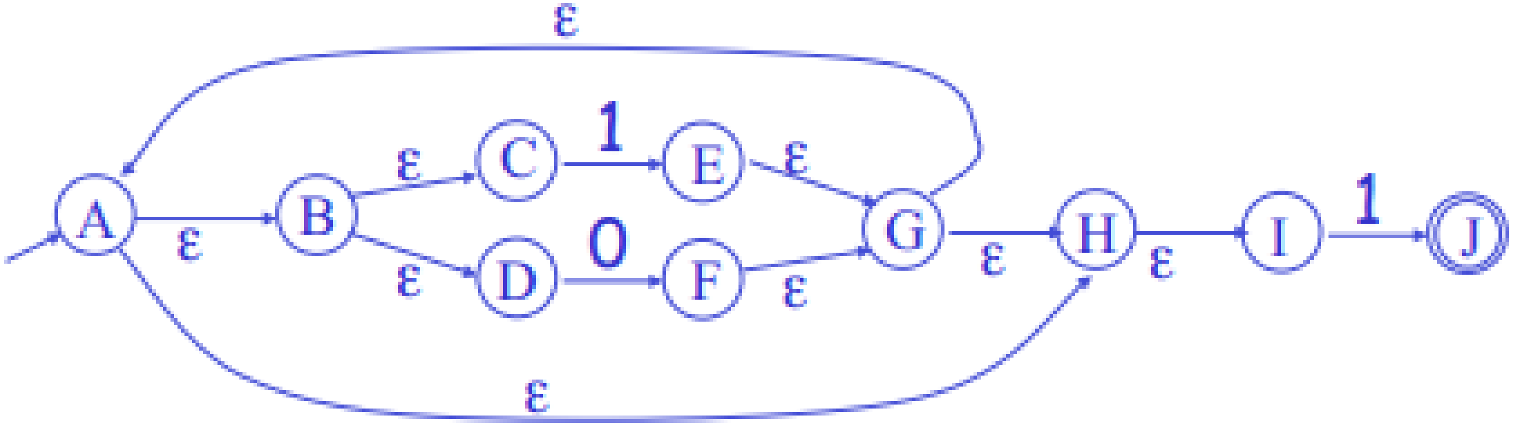


# Example of RegExp -> NFA conversion

- Consider the regular expression

  (1+0)*1

- The NFA is



# ε-closure of a state

- ε-closure of a state s is a set of states that consists of s and all other states that I can reach from s by making ε-moves only.
- Example
  - ε-closure(B) = {B, C, D}
  - ε-closure(G) = {G, H, I, A, B, C, D}

# NFA to DFA. Remark

- An NFA may be in many states at any time
- How many different states ?
- If there are N states, the NFA must be in some subset of those N states
- How many subsets are there?
  - $2^N$-1 i.e., finitely many

| NFA | DFA |
|---|---|
| States : S | States : subset of S |
| Start state: $s \in S$ | Start state: $\epsilon$-closure(s) |
| Final states: F subset of S | Final state: $\{ X \mid X \cap F \neq \phi \}$ |
| The transition function: $a(x)=\{y \mid x \in X \wedge x \xrightarrow{a} y\}$ | The transition function: $X \xrightarrow{a} Y$ if $Y= \epsilon\text{-closure}(a(X))$ |

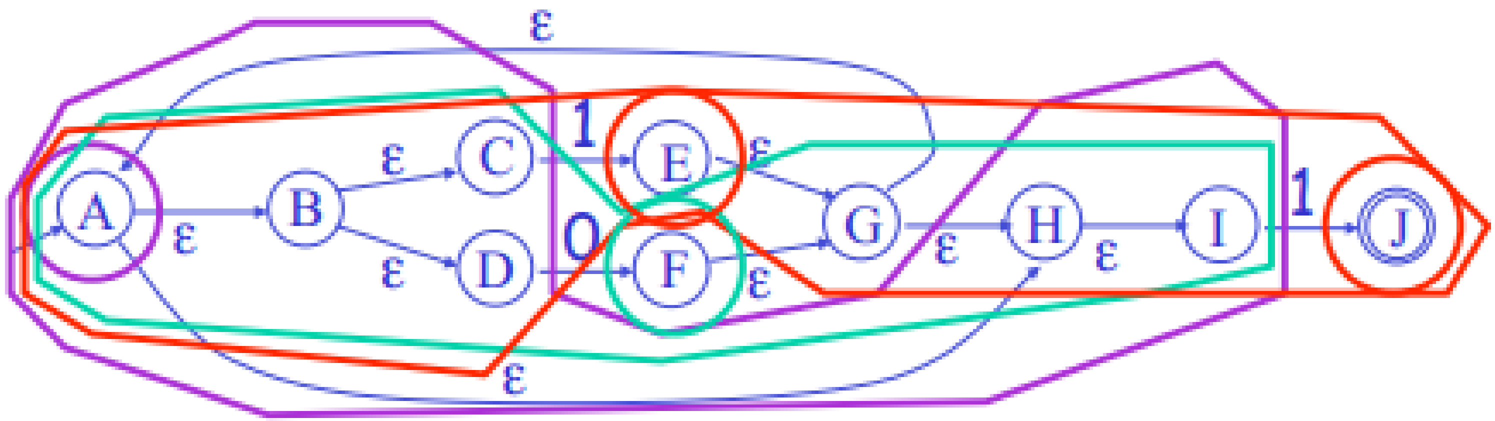
# NFA to DFA: The Trick

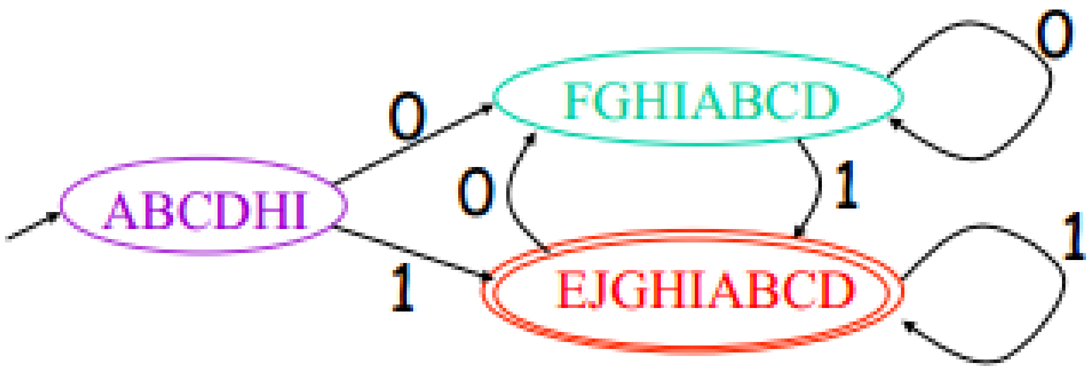
- Simulate the NFA
- Each state of DFA

  = a non-empty subset of states of the NFA
- Start state

  = the set of NFA states reachable through ε-moves from NFA start state
- Add a transition $S \rightarrow^{a} S'$ to DFA iff
  - $S'$ is the set of NFA states reachable from any state in $S$ after seeing the input a, considering ε-moves as well

# NFA -> DFA Example

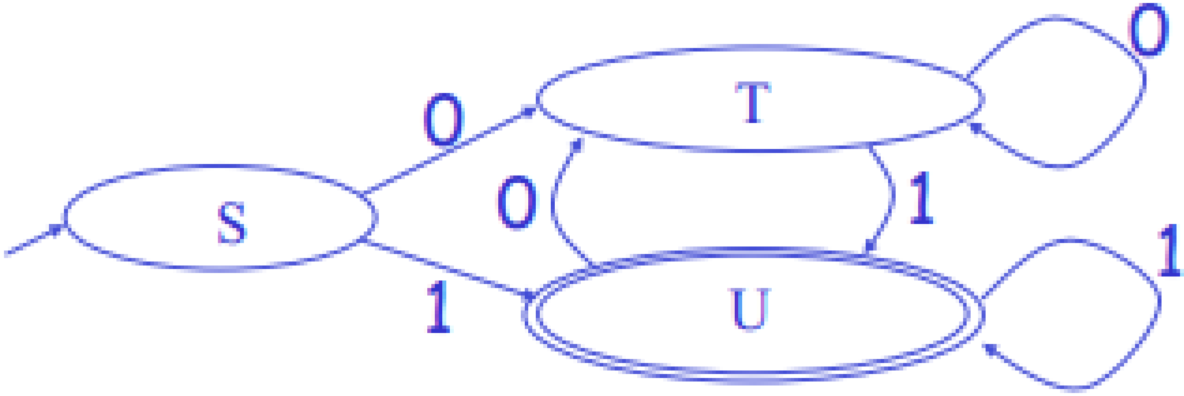


# Implementation

- A DFA can be implemented by a 2D table T
  - One dimension is “states”
  - Other dimension is “input symbol”
  - For every transition $S_i \rightarrow^a S_k$ define T[i,a] = k
- DFA “execution”
  - If in state $S_i$ and input a, read T[i,a] = k and skip to state $S_k$
  - Very efficient

# Table Implementation of a DFA

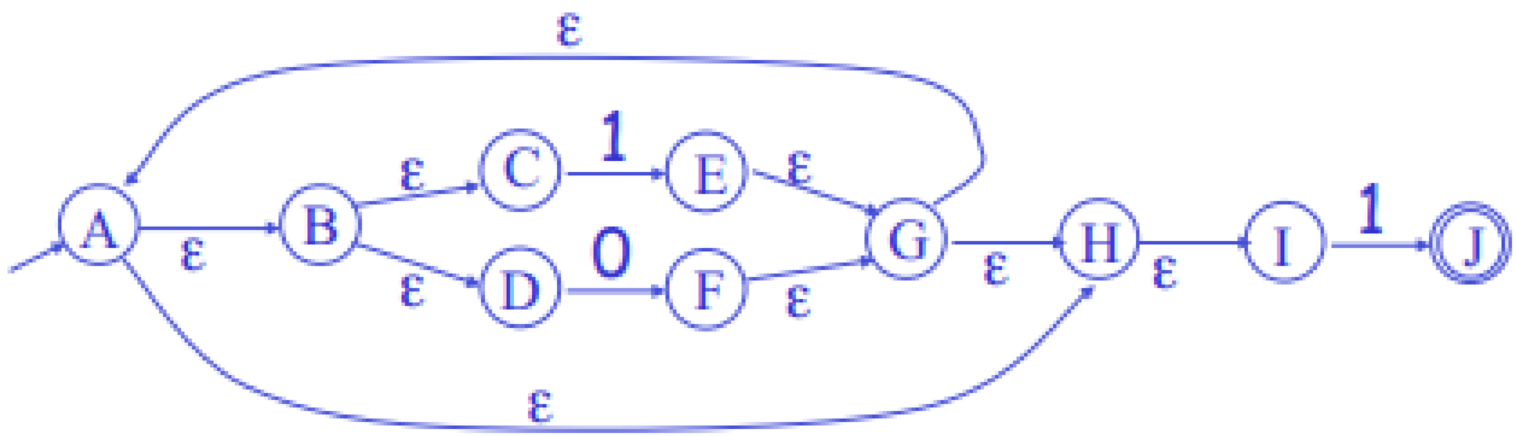



| | 0 | 1 |
|---|---|---|
| S | T | U |
| T | T | U |
| U | T | U |

# algorithm

```
i=0;
State=0;
While(input[i]){
    State=A[state, input[i++]];
    }
```

# Implementation of the NFA itself




| | 0 | 1 | ξ |
|---|---|---|---|
| A | | | **{B}** |
| B | | | { C,D } |
| C | | { E } | |
| D | { F } | | |
| : | | | |

# Trade off between speed and space

- DFAs
  - Faster: we are in one state at any given time.
  - less compact: there could be a large number of states $2^N$-1.
- NFAs
  - slower  (the loop has to deal with set of states rather than one state),
  - concise