GENERAL MATHEMATICS 2

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September 1, 2020

Chapter 1: CONIC SECTIONS
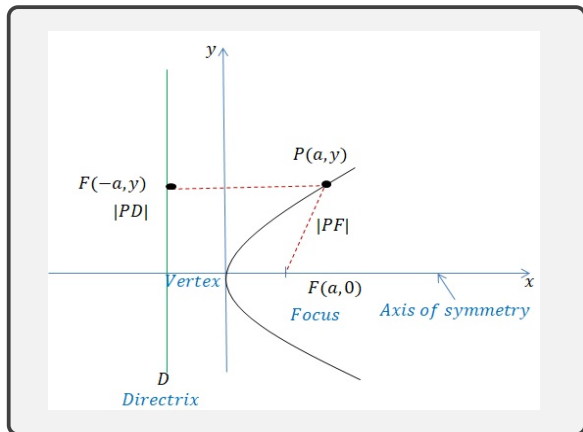
Main Contents

1. Parabola
2. Ellipse
3. Hyperbola

Section 1: Parabola

Definition

A parabola is a set of all points in a plane that are equidistant from a fixed point F (called the focus) and a fixed line D (called the directrix) in the same plane.
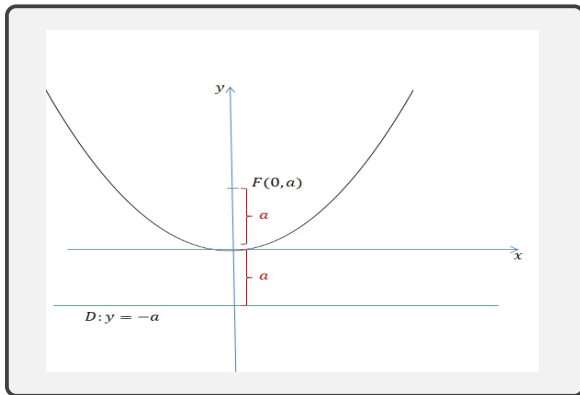
Section 1: Parabola

(1) Parabolas with the Vertex at the Origin

(A) Vertical Parabolas

The equation of the vertical parabola with the vertex at the origin is $x^2 = \pm 4ay$, where $a > 0$.

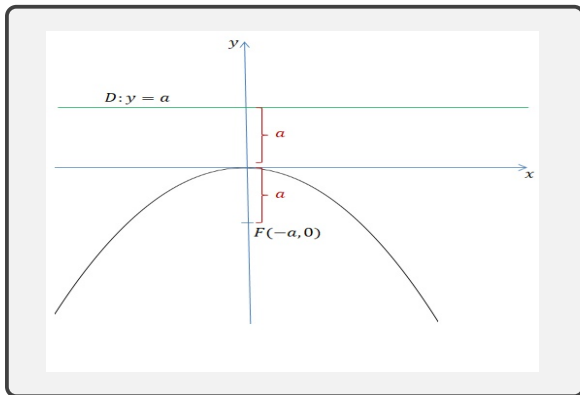**(A.1)** The equation $x^2 = 4\,a\,y$ has the following properties:

- The vertex of the parabola is at the origin $V(0,0)$.

- The parabola opens upwards.

- The axis of symmetry of the parabola is y-axis.

- The focus of the parabola is $F(0,a)$.

- The directrix of the parabola is $D : y = -a$.

Section 1: Parabola

(A.2) The equation $x^2 = -4\,a\,y$ has the following properties:

- The vertex of the parabola is at the origin $V(0, 0)$.
- The parabola opens downwards.
- The axis of symmetry of the parabola is y-axis.
- The focus of the parabola is $F(0, -a)$.
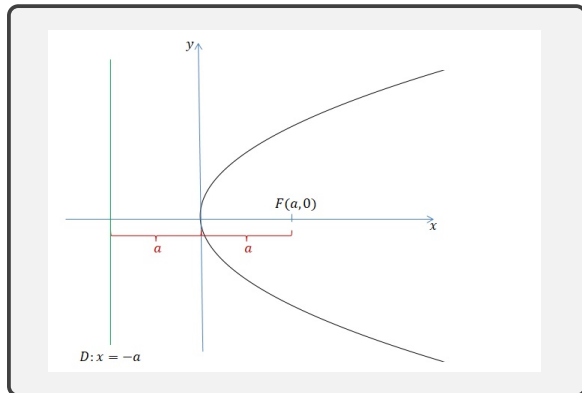- The directrix of the parabola is $D : y = a$.

Section 1: Parabola

(B) Horizontal Parabolas

The equation of the horizontal parabola with the vertex at the origin is $y^2 = \pm 4\,a\,x$, where $a > 0$.

(B.1) The equation $y^2 = 4\,a\,x$ has the following properties:

- The vertex of the parabola is at the origin $V(0,0)$.
- The parabola opens to the right.
- The axis of symmetry of the parabola is x-axis.
- The focus of the parabola is $F(a,0)$.
- The directrix of the parabola is $D : x = -a$.

Section 1: Parabola

(B.2) The equation $y^2 = -4\,a\,x$ has the following properties:
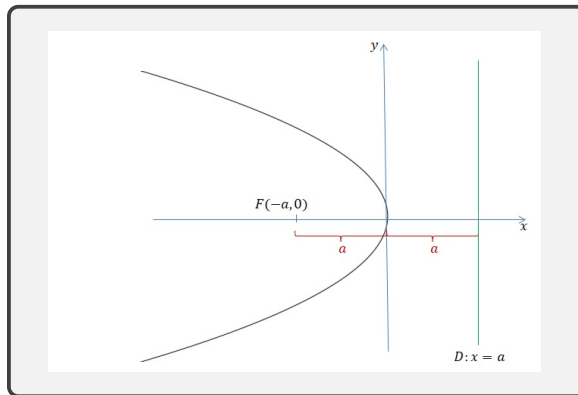
- The vertex of the parabola is at the origin $V(0, 0)$.
- The parabola opens to the left.
- The axis of symmetry of the parabola is x-axis.
- The focus of the parabola is $F(-a, 0)$.
- The directrix of the parabola is $D : x = a$.

Section 1: Parabola

Example

Find the focus and the directrix of the parabola $x^2 = 4y$, and sketch its graph.

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Find the focus and the directrix of the parabola $x^2 = 4y$, and sketch its graph.
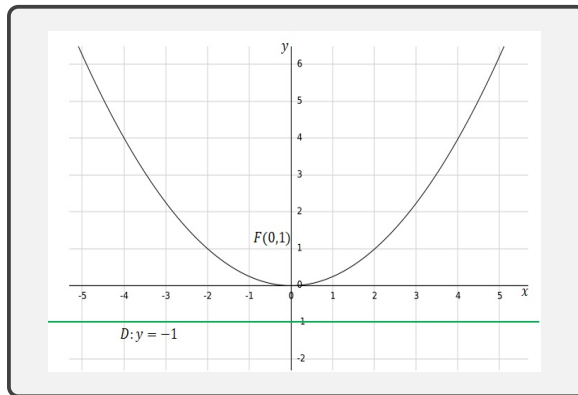
Solution:
The equation $x^2 = 4y$ takes the form
$x^2 = 4 a y$

$$\Rightarrow 4a = 4 \Rightarrow a = 1$$

Therefore, the parabola has the following properties:

- The vertex of the parabola is $V(0,0)$.
- The parabola opens upwards.
- The axis of symmetry of the parabola is y-axis.
- The focus of the parabola is $F(0,1)$.
- The directrix of the parabola is $D : y = -1$.

Section 1: Parabola

Example

Find the focus and the directrix of the parabola $y^2 = -8x$, and sketch its graph.

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Find the focus and the directrix of the parabola $y^2 = -8x$, and sketch its graph.

Solution:
The equation $y^2 = -8x$ takes the form $y^2 = -4\,a\,x$

$$\Rightarrow 4a = 8 \Rightarrow a = 2$$

The parabola has the following properties:

- The vertex of the parabola is $V(0, 0)$.
- The parabola opens to the left.
- The axis of symmetry of the parabola is x-axis.
- The focus of the parabola is $F(-2, 0)$.
- The directrix of the parabola is $D : x = 2$.