Ch 4

Lecture 8

Characteristics of actuarial models

parametric and scale distribution

Defn ①

A parametric distribution is a set of distribution functions each member of which is determined by specifying one or more values called parameters. the number of parameters is fixed and finite.

Defn ②

A parametric distribution is a scale distribution if when a random variable from that set of distributions is multiplied by a positive constant, the resulting random variable is also in that set of distributions.

Ex 4.1 p.51

Demonstrate that the exponential distribution is a scale distribution

Ans:

For the exponential distn, the distn fn of $X \sim exp(\theta)$ is

$$F_X(x) = 1 - e^{-x/\theta} \quad , \text{see } p.469$$

let $Y = cX$, where $c > 0$. Then,

$$F_Y(y) = pr\left(X \le \frac{y}{c}\right)$$

$$\therefore F_Y(y) = 1 - e^{-y/c\theta}$$

this is an exponential distn with parameter $c\theta$

$\therefore \theta$ is a scale parameter.

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EX 4.2 p.51
Demonstrate that the gamma distribution, as defined in Appendix A, has a scale parameter.

Ans:

$$\because F_X(x) = \Gamma(\alpha; x/\theta)$$ See, p. 468

$$\Rightarrow F_Y(y) = pr(X \le \frac{y}{c}), \text{ where } Y = cX, \ c > 0$$

$$= \Gamma(\alpha; \frac{y}{c\theta})$$

$\therefore$ Y has a gamma distn with parameters α and $c\theta$.

$\therefore$ θ is a scale parameter. #

Ex 4.1 p.56) Demonstrate that the lognormal distribution as parameterized in Appendix A is a scale distribution but has no scale parameter. Display an alternative parameterization of this distribution that does have a scale parameter.

Ans:

As before, For $F_Y(y) = pr(X \le y/c)$

$$= \Phi\left(\frac{\ln(y/c) - \mu}{\sigma}\right)$$

$$F_Y(y) = \Phi\left[\frac{\ln y - (\ln c + \mu)}{\sigma}\right]$$

$$\Longrightarrow \quad Y \sim \text{lognormal}(\mu + \ln c, \sigma)$$

i.e there's no parameter was multiplied by c, there is no scale parameter.

* To introduce a scale parameter, let's define the lognormal distn fn as

$$F(x) = \Phi\left(\frac{\ln x - \ln \upsilon}{\sigma}\right),$$

with parameters υ, σ, where $\upsilon = e^{\mu}$

$$\Rightarrow F_Y(y) = pr(X \leq y/c), \text{ when } Y = cX,\ c > 0$$

$$= \Phi\left[\frac{\ln(y/c) - \ln \upsilon}{\sigma}\right]$$

$$= \Phi\left[\frac{\ln y - (\ln c + \ln \upsilon)}{\sigma}\right]$$

$$F_Y(y) = \Phi\left[\frac{\ln y - \ln c\upsilon}{\sigma}\right]$$

which indicates that Y has a lognormal dist^n with parameters $c\upsilon, \sigma$ when υ is a scale parameter.

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