Lecture (3)

Defn (7) معدل الخطر (معدل الوفيات) معدل الفشل (الأعطال)

The hazard rate, also known as the force of mortality and the failure rate is defined as

$$h(x) = h_X(x) = f_X(x) / S_X(x)$$

$$\Rightarrow h_X(x) = -S_X'(x) / S_X(x)$$

$$h_X(x) = \frac{-d}{dx}\left[\ln S_X(x)\right]$$

$$\Rightarrow \int_0^b d\left[\ln S_X(x)\right] = -\int_0^b h(x)\,dx$$

$$\therefore \ln S(b) - \ln S(0) = -\int_0^b h(x)\,dx$$

$$\ln S(b) - \ln(1) = -\int_0^b h(x)\,dx$$

$$\therefore S(b) = e^{-\int_0^b h(x)\,dx}$$

For our Four Models

Model (1)

$$h_1(x) = 0.01/(1-0.01x) = \frac{0.01}{1-0.01x}, \quad 0 < x < 100$$

See fig (2.7) p.18

Model (2)

$$h_2(x) = \frac{3(2{,}000)^3}{(x+2{,}000)^4} \Big/ \left(\frac{2{,}000}{x+2{,}000}\right)^3$$

$$\therefore h_2(x) = \frac{3}{x+2{,}000}, \quad x > 0$$

See fig (2.8) p.18

Model ③

The hazard rate for Model ③ $h_3(x)$ is not defined (the hazard rate is only defined for prob. density fn See Defn ⑦

Model ④

$$h_4(x) = \frac{0.000003\, e^{-0.00001x}}{0.3\, e^{-0.00001x}}$$

$$h_4(x) = \frac{3\times10^{-6}}{3\times10^{-1}} = 10^{-5}$$

$$\therefore h_4(x) = 0.00001, \quad x > 0$$

Note that: For the mixed dists in Model ④, the hazard rate is only defined over $(0, \infty)$ not on $[0, \infty)$.

EX: Model ⑤ p.19

Consider the lifetime distribution defined as follows:

$$f_5(x) = \begin{cases} 1 - 0.01x, & 0 \le x < 50 \\ 1.5 - 0.02x, & 50 \le x < 75 \end{cases}$$

Determine the distribution, density and hazard rate functions for Model ⑤, Construct graphs for all of these functions.

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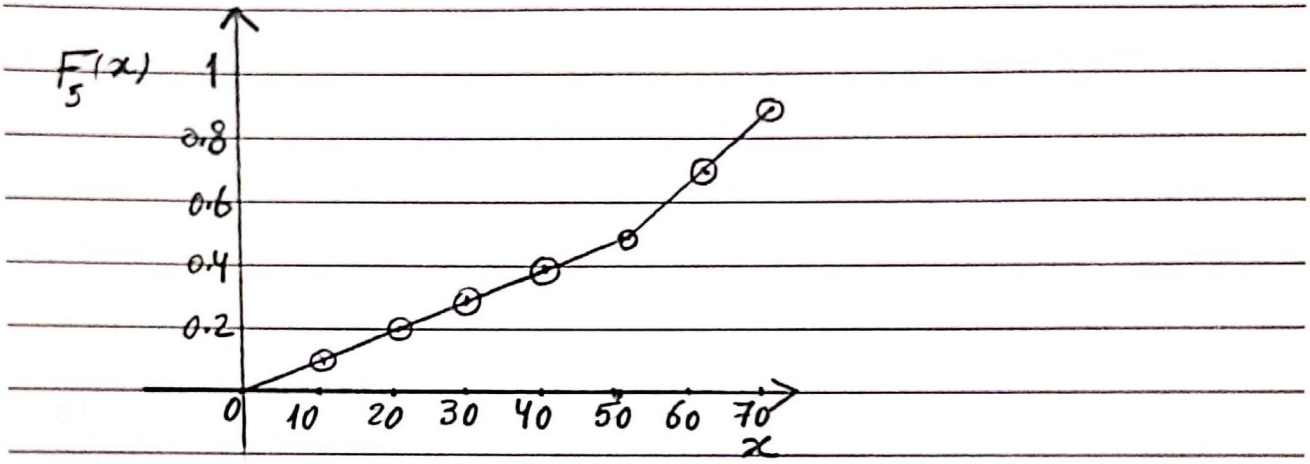
* The distn fn for Model ⑤

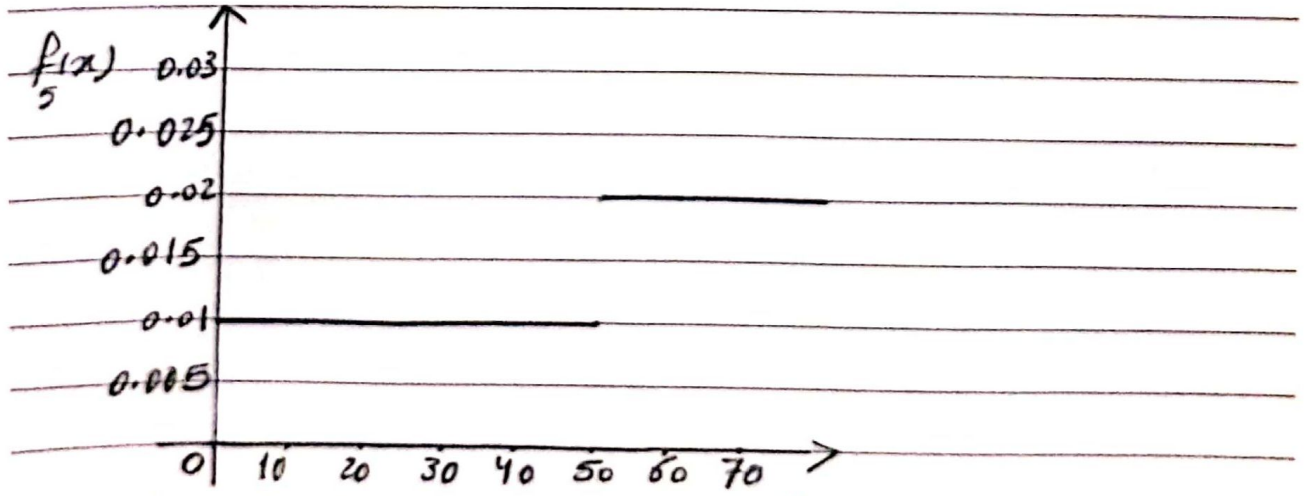
$$\Rightarrow F_5(x) = \begin{cases} 0.01x, & 0 \le x < 50 \\ 0.02x - 0.5, & 50 \le x < 75 \end{cases}$$



* The density fn for Model ⑤

$$\Rightarrow f_5(x) = -5'(x)$$

$$= \begin{cases} 0.01, & 0 \le x < 50 \\ 0.02, & 50 \le x < 75 \end{cases}$$



Note that: the density fn $f_5(x)$ is not defined at 50.

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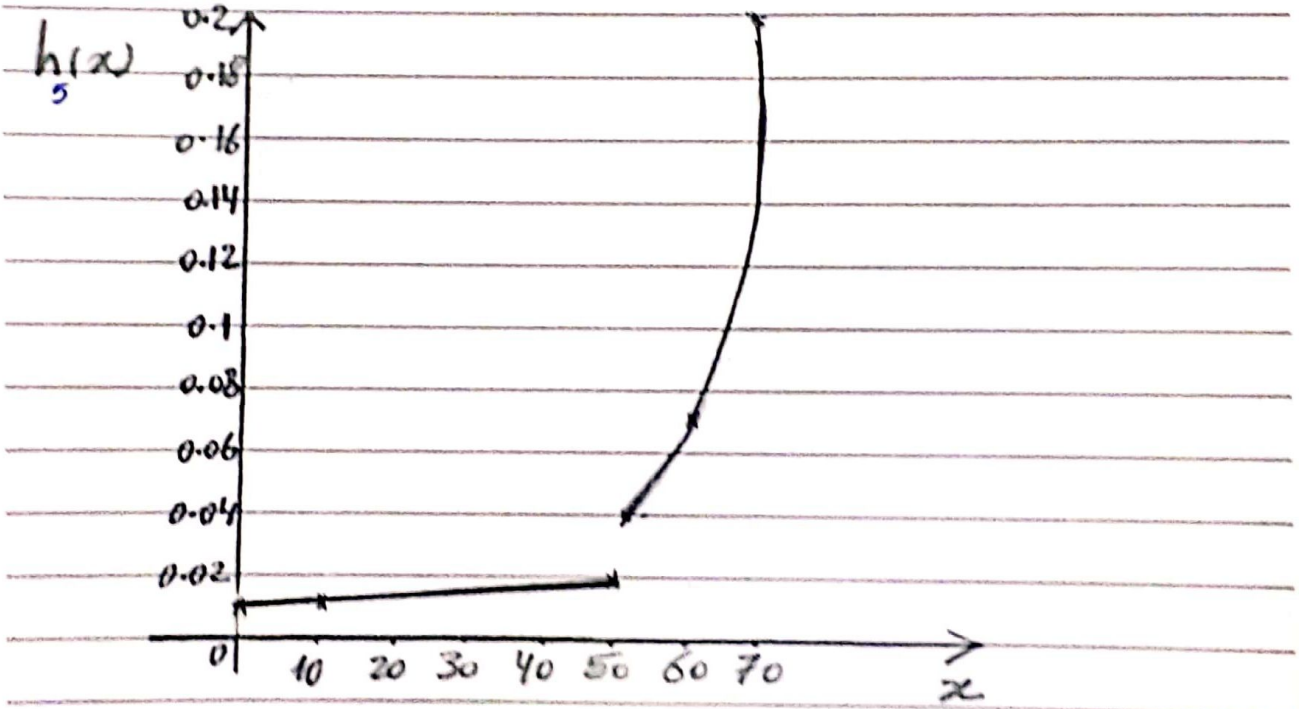
* the hazard fn for Model ⑤

$$h(x) = \frac{f(x)}{S(x)}$$

$$h_5(x) = \begin{cases} \frac{0.01}{1-0.01x}, & 0 \le x < 50 \\ \frac{0.02}{1.5-0.02x}, & 50 \le x < 75 \end{cases}$$

$$h_5(x) = \begin{cases} \frac{1}{100-x}, & 0 \le x < 50 \\ \frac{1}{75-x}, & 50 \le x < 75 \end{cases}$$



at 50, $h_5(50) = \frac{1}{100-50} = 0.02$, also $h_5(50) = \frac{1}{75-50} = 0.04$

Note that: the hazard fn is not defined at 50.
i.e (not continuous at 50)

• Defn ⑧

The mode of a random variable is the most likely value. It's the value with the largest prob. fn / density fn.