



Lecture (27)
Ch 16: Credibility p. 392 Textbook

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EX 16.2 p. 392

Ans

* Case (1) Accuracy is to be measured with regard to the average number of claims. Then, using the N_j 's rather than the X_j 's, we have $\bar{J} = E(N_j) = \lambda$ and $\sigma^2 = \text{Var}(N_j) = \lambda$

\therefore We have $n \geq \lambda_0 \left(\frac{\sigma}{\bar{J}}\right)^2$ for full credibility

$$\Rightarrow n \geq \lambda_0 \frac{\sigma^2}{\bar{J}^2} = \lambda_0 \left(\frac{\lambda}{\lambda^2}\right)$$

$$\therefore n \geq \frac{\lambda_0}{\lambda}$$

λ_0 is estimated before in EX 16.1, $\lambda_0 = 1082.41$

For the given data, we have 5 claims, so $\lambda = \frac{5}{10} = 0.5$ per policy

$$\therefore n \geq \frac{1082.41}{0.5} = 2164.82$$

clearly, the 10 policies are far short of this standard.

* Case (2):

When accuracy is with regard to the average total payment, we have $\bar{J} = E(X_j) = \lambda \theta_Y$ and $\text{Var}(X_j) = \lambda(\sigma_Y^2 + \sigma^2)$

See ch 9

Again, \therefore We have $n \geq \lambda_0 \left(\frac{\sigma}{\bar{J}}\right)^2$ for full credibility

$$\Rightarrow n \geq \lambda_0 \frac{\lambda(\sigma_Y^2 + \sigma^2)}{\lambda^2 \theta_Y^2} = \frac{\lambda_0}{\lambda} \left[1 + \left(\frac{\sigma_Y}{\theta_Y}\right)^2\right]$$

For the given data:

253 398 439 129 627

\Rightarrow The mean $\theta_Y = 369.2$ and Variance (for 5 claims) is

$$\sigma_Y^2 = \frac{\sum (x_j - 369.2)^2}{4} = 35840.2$$

$$\Rightarrow \sigma_Y = 189.315$$

$$\therefore n \geq \frac{1082.41}{0.5} \left[1 + \left(\frac{189.315}{369.2}\right)^2\right] = 2734.02$$

clearly, the 10 observations are far short of what is needed.



Note that:

$$\frac{10}{2,734.02} = \frac{5}{1,367.01} = \frac{1846}{504,701} = 0.003658$$

$\xrightarrow{\times 0.5}$ $\xrightarrow{\times 369.2}$

where, $253 + 398 + 439 + 129 + 627 = 1846$ (total payment of 5 claims).

* Clearly, the standard for full credibility is not suitable method (inappropriate). The partial credibility through credibility premium, it will be more appropriate method, see, Lecture (28).

EX (16.4) p. 395

Suppose, in EX 16.2, that the manual premium M is 225. Determine the credibility estimate using both cases.

Ans

* For the first case, the credibility factor is

$$Z = \sqrt{\frac{n}{n + \sigma^2}} = \sqrt{\frac{10}{2164.82}}$$

$$Z = 0.06797$$

$$\rightarrow P_c = Z\bar{X} + (1 - Z)M$$

$$P_c = 0.06797(184.6) + 0.93203(225) = 222.25$$

where the average of the payments is $\bar{X} = \frac{1846}{10} = 184.6$ and the manual premium M is 225.

* For the second case, we can use any of the three calculations:

$$Z = \sqrt{\frac{10}{2734.02}} = \sqrt{\frac{5}{1367.01}} = \sqrt{\frac{1846}{504701}} = 0.06048$$

then

$$P_c = 0.06048(184.6) + 0.93952(225) = 222.56$$

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