1/

لا يكتب في هذا الهامش

Lecture 26

Ch 16: Introduction to limited fluctuation credibility

*EX 16.1 p.391

Suppose that past losses $X_1, \dots, X_n$ are available for a particular policyholder. The sample mean is to be used to estimate $\xi = E(X_j)$. Determine the standard for full credibility. Then suppose that there were 10 observations, with six being zero and the others being 253, 398, 439 and 756. Determine the full-credibility standard for this case with $r = 0.05$ and $p = 0.9$.

Ans:

For full-credibility, $n \geq \lambda_0 \left(\frac{\sigma}{\xi}\right)^2$ (1)

As before $\Phi(y_p) = (1+p)/2$

at $p = 0.9 \Rightarrow \Phi(y_p) = 0.95$

$\Rightarrow y_p = 1.645$ (calculated from standard Normal Table)

$\because \lambda_0 = (y_p/r)^2 \therefore \lambda_0 = \left(\frac{1.645}{0.05}\right)^2 = 1082.41$ (2)

The mean is $\xi = E(X_j)$

$$= \frac{0 + 253 + 398 + 439 + 756}{10} = 184.6 \quad (3)$$

and variance is

$$\sigma^2 = \sum_j (x_j - \xi)^2 / (n-1) \quad \text{(For unbiased estimate)}$$

$$\sigma^2 = \frac{6(184.6)^2 + (253 - 184.6)^2 + (398 - 184.6)^2 + (439 - 184.6)^2 + (756 - 184.6)^2}{9}$$

$\sigma^2 = 71766.48889 \Rightarrow \sigma = 267.89$ (4)

Substitute (2), (3), (4) in (1), we get

$$n \geq 1082.41 \left(\frac{267.89}{184.6}\right)^2 = 2279.51$$

$\therefore$ The 10 observations do not deserve full credibility.

#

2//

* partial credibility p.393

In partial credibility, we can combine the past experience $\bar{X}$, with the credibility premium M as follows:

$$P_c = Z\bar{X} + (1-Z)M \qquad (1)$$

where $Z = \frac{n}{n+k}$, k needs to be determined

or

$$Z = (\xi/\sigma)\sqrt{n/\lambda_0} \qquad \text{(See p. 394)}$$

i.e $Z = \sqrt{\frac{n}{\lambda_0 \sigma^2/\xi^2}}$ ... which is called square root rule. (2)

*EX 16.3 p.394

Suppose in Example 16.1 that the manual premium M is 225. Determine the credibility estimate

Ans: The average of the payments is $\bar{X} = 184.6$

(2) → $Z = \sqrt{\frac{n}{\lambda_0 \sigma^2/\xi^2}} = \sqrt{\frac{10}{1082.41\left(\frac{267.89}{184.6}\right)^2}}$

$$Z = \sqrt{\frac{10}{2279.51}} = 0.06623$$

see EX 16.1

Then, the credibility premium (partial credibility) is

$$P_c = Z\bar{X} + (1-Z)M$$

$$P_c = 0.06623(184.6) + 0.93377(225)$$

$$\therefore P_c = 222.32$$

//

3

*EX 16.5 p. 395

Ans:

For the group which has past experience, we have

① $n = 100 + 110 = 210$ observations (# of exposure units)

② The average $\bar{X} = 150$ of 210 claims, and we can assume that $\sigma = 140$ for this group.

For full credibility, $n \geq \lambda_0 \left(\frac{\sigma}{\delta}\right)^2$

λ_0 was calculated before in EX (16.1) at $p = 0.9$ and $r = 0.05$

$\Rightarrow \lambda_0 = 1082.41$

$\Rightarrow n \geq 1082.41 \left(\frac{140}{150}\right)^2 = 942.9$

$\Rightarrow Z = \sqrt{\frac{n}{\lambda_0 \sigma^2 / \delta^2}} = \sqrt{\frac{210}{942.9}}$

$Z = 0.4719$

Thus, the net premium per person insured is

$P_c = Z\bar{X} + (1-Z)M, \quad M = 175$

$= 0.4719(150) + 0.5281(175)$

$\therefore P_c = 163.2$

$\Rightarrow$ The net premium for 125 lives will be insured next year $= 125(163.2)$

$\simeq 20{,}400$