

Lecture (25)

Ch 16: Introduction to limited fluctuation credibility (p. 387)

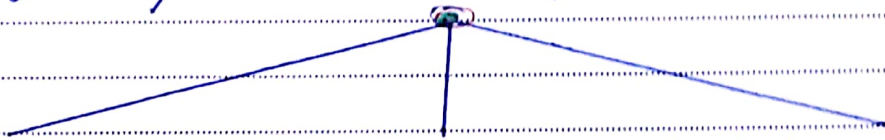
Suppose that a policyholder has experienced X_j claims or losses in a past experience period j , where $j \in \{1, 2, 3, \dots, n\}$

$$\text{let } E(X_j) = \bar{J}, \quad \text{Var}(X_j) = \sigma^2$$

⇒ The average $\bar{X} = n^{-1}(X_1 + X_2 + \dots + X_n)$, $E(\bar{X}) = \bar{J}$

For independent X_j , $\text{Var}(\bar{X}) = \sigma^2/n$

In credibility theory, we have three possibilities for the value \bar{J} .



The first, is to ignore the past data (no credibility) and simply charge M . M is called the manual premium.

The second, is to ignore M and charge \bar{X} (full credibility).

The third, is to choose some combination of M and \bar{X} (partial credibility).

Note that: From the insurer's standpoint, if the experience is more stable (less variation in data, σ^2 small), then \bar{X} is of more use as a predictor of next year's results. Conversely, if the experience is more volatile (more variation), then \bar{X} is of less use as a predictor of next year's results and the choice of M makes more sense.

① Full credibility

To infer that \bar{X} is stable, let $\bar{X} - \bar{J}$ be small with high probability

$$\Rightarrow \text{let } \text{pr}(-r\bar{J} \leq \bar{X} - \bar{J} \leq r\bar{J}) \geq p \quad (1)$$

where $r = 0.05$ and $p = 0.9$

$$\Rightarrow \text{pr}\left(\left|\frac{\bar{X} - \bar{J}}{\sigma/\sqrt{n}}\right| \leq \frac{r\bar{J}\sqrt{n}}{\sigma}\right) \geq p \quad (2)$$

* The condition for full credibility is

$$\frac{r \bar{y} \sqrt{n}}{\sigma} \geq y_p \quad (3)$$

where y_p satisfies that

$$\Pr\left(\left|\frac{\bar{X} - \bar{y}}{\sigma/\sqrt{n}}\right| \leq y_p\right) = p \quad (4)$$

See p. (3)

$$(3) \Rightarrow \frac{\sigma}{\bar{y}} \leq \frac{r\sqrt{n}}{y_p} \quad (5)$$

$$\Rightarrow \frac{\sigma}{\bar{y}} \leq \sqrt{\frac{n}{\lambda_0}}, \quad \lambda_0 = (y_p/r)^2 \quad (6)$$

i.e. the full credibility is occurred if the coefficient of variation σ/\bar{y} is no larger than $\sqrt{n/\lambda_0}$

Also, full credibility occurs when $\text{Var}(\bar{X}) = \frac{\sigma^2}{n} \leq \frac{\bar{y}^2}{\lambda_0}$ as shown in the following:

$$(6) \Rightarrow \sigma \leq \bar{y} \sqrt{\frac{n}{\lambda_0}}$$

$$\sigma^2 \leq \bar{y}^2 \frac{n}{\lambda_0}$$

$$\therefore \text{Var}(\bar{X}) = \frac{\sigma^2}{n} \leq \frac{\bar{y}^2}{\lambda_0} \quad (7)$$

For n that gives the number of exposure units required for full credibility,

$$(6) \Rightarrow \frac{\sigma^2}{\bar{y}^2} \leq \frac{n}{\lambda_0}$$

$$\Rightarrow n \geq \lambda_0 \left(\frac{\sigma}{\bar{y}}\right)^2 \quad (8)$$

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Notes

$$\text{If } \bar{X} \sim N(\bar{\mu}, \sigma^2/n)$$

By using limit theorem for large n

$$Z = \frac{\bar{X} - \bar{\mu}}{\sigma/\sqrt{n}} \sim N(0,1) \quad \text{Standard Normal distn}$$

$$\begin{aligned} \textcircled{4} \Rightarrow P &= \text{pr}(|Z| \leq y_p) \\ &= \text{pr}(-y_p \leq Z \leq y_p) \\ &= \Phi(y_p) - \Phi(-y_p) \\ &= \Phi(y_p) - [1 - \Phi(y_p)] \end{aligned}$$

$$\therefore P = 2\Phi(y_p) - 1$$

$\Rightarrow \Phi(y_p) = (1+P)/2$ and y_p is the $(1+P)/2$ percentile of the standard normal distn.

For $P=0.9$ and $\Gamma=0.05$

$$\Rightarrow \Phi(y_p) = 0.95$$

$y_p = 1.645$ (calculated from standard normal table)

$$\textcircled{6} \Rightarrow \lambda_0 = (1.645/0.05)^2 = 1082.41$$

$$\textcircled{8} \Rightarrow n \geq \lambda_0 \left(\frac{\sigma}{\bar{\mu}}\right)^2$$

$$\therefore n \geq 1082.41 \frac{\sigma^2}{\bar{\mu}^2}$$

where the quantity $\frac{\sigma}{\bar{\mu}}$ is called the coefficient of variation.

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