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Lecture (25)

Ch 16: Introduction to limited fluctuation credibility (p. 387)

Suppose that a policyholder has experienced X_j claims or losses in a past experience period j, where $j \in \{1, 2, 3, \ldots, n\}$

$$\text{let } E(X_j) = \xi, \quad Var(X_j) = \sigma^2$$

$\Rightarrow$ The average $\bar{X} = n^{-1}(X_1 + X_2 + \cdots + X_n)$, $E(\bar{X}) = \xi$

For independent X_j, $Var(\bar{X}) = \sigma^2/n$

In credibility theory, we have three possibilities for the value ξ

The first, is to ignore the past data (no credibility) and simply charge M, M is called the manual premium

The second, is to ignore M and charge $\bar{X}$ (full credibility)

The third is to choose some combination of M and $\bar{X}$ (partial credibility)

Note that: from the insurer's standpoint, if the experience is more stable (less variation in data, σ^2 small), then $\bar{X}$ is of more use as a predictor of next year's results. Conversely, if the experience is more volatile (more variation), then $\bar{X}$ is of less use as a predictor of next year's results and the choice of M makes more sense.

I * Full credibility

To infer that $\bar{X}$ is stable, let $\bar{X} - \xi$ be small with high probability

$$\Rightarrow \text{let } pr(-r\xi \leq \bar{X} - \xi \leq r\xi) \geq p \qquad (1)$$

where $r = 0.05$ and $p = 0.9$

$$\Rightarrow pr\left(\left|\frac{\bar{X} - \xi}{\sigma/\sqrt{n}}\right| \leq \frac{r\xi\sqrt{n}}{\sigma}\right) \geq p \qquad (2)$$

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* The condition for full credibility is

$$\frac{r\,\xi\sqrt{n}}{\sigma} \geq y_p \qquad (3)$$

where y_p satisfies that

$$pr\left(\left|\frac{\bar{X}-\xi}{\sigma/\sqrt{n}}\right| \leq y_p\right) = p \qquad (4)$$

see p. 3

$$(3) \Rightarrow \frac{\sigma}{\xi} \leq \frac{r\sqrt{n}}{y_p} \qquad (5)$$

$$\Rightarrow \frac{\sigma}{\xi} \leq \sqrt{\frac{n}{\lambda_0}}, \quad \lambda_0 = (y_p/r)^2 \qquad (6)$$

i.e the full credibility is occurred if the coefficient of variation σ/ξ is no larger than $\sqrt{n/\lambda_0}$

Also, full credibility occurs when $Var(\bar{X}) = \frac{\sigma^2}{n} \leq \frac{\xi^2}{\lambda_0}$ as shown in the following:

$$(6) \Rightarrow \sigma \leq \xi\sqrt{\frac{n}{\lambda_0}}$$

$$\sigma^2 \leq \frac{\xi^2 n}{\lambda_0}$$

$$\therefore Var(\bar{X}) = \frac{\sigma^2}{n} \leq \frac{\xi^2}{\lambda_0} \qquad (7)$$

For n that gives the number of exposure units required for full credibility,

$$(6) \Rightarrow \frac{\sigma^2}{\xi^2} \leq \frac{n}{\lambda_0}$$

$$\Rightarrow n \geq \lambda_0\left(\frac{\sigma}{\xi}\right)^2 \qquad (8)$$

Note:

If $\bar{X} \sim N(\xi, \sigma^2/n)$

By using limit theorem for large n

$$Z = \frac{\bar{X} - \xi}{\sigma/\sqrt{n}} \sim N(0,1)$$

Standard Normal distn

$$④ \Rightarrow P = pr(|Z| \leq y_p)$$

$$= pr(-y_p \leq Z \leq y_p)$$

$$= \Phi(y_p) - \Phi(-y_p)$$

$$= \Phi(y_p) - [1 - \Phi(y_p)]$$

$$\therefore P = 2\Phi(y_p) - 1$$

$\Rightarrow$ $\Phi(y_p) = (1+P)/2$ and y_p is the $(1+P)/2$ percentile of the standard normal distn.

For $P = 0.9$ and $r = 0.05$

$\Rightarrow \Phi(y_p) = 0.95$

$\therefore y_p = 1.645$ (calculated from standard normal table)

⑥ $\Rightarrow n_0 = (1.645/0.05)^2 = 1082.41$

⑧ $\Rightarrow n \geq n_0 \left(\frac{\sigma}{\xi}\right)^2$

$$\therefore n \geq 1082.41 \frac{\sigma^2}{\xi^2}$$

where the quantity $\frac{\sigma}{\xi}$ is called the coefficient of variation.

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