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Lecture 24
Mathematical Statistics
Ch 11 p.242 - p.241
Textbook

*pb 11.31 p.242

Three losses are observed with value 66, 91, and 186. Seven other losses are known to be less than or equal to 60. Losses have an inverse exponential distribution with cdf $F(x) = e^{-\theta/x}$, $x > 0$. Determine the maximum likelihood estimate of the population mode.

Ans:

The pdf for inv. exp. distn is $f(x) = \theta x^{-2} e^{-\theta/x}$

The likelihood fn

$$\Rightarrow L(\theta) = \theta(66)^{-2} e^{-\theta/66} \theta(91)^{-2} e^{-\theta/91} \theta(186)^{-2} e^{-\theta/186} (e^{-\theta/60})^7$$
$$\propto \theta^3 e^{-0.148184\theta}$$

$$\therefore \ell(\theta) = 3\ln\theta - 0.148184\theta$$
$$\ell'(\theta) = \frac{3}{\theta} - 0.148184$$

To get $\hat{\theta}$, set $\ell'(\theta) = 0$

$$\frac{3}{\theta} - 0.148184 = 0 \Rightarrow \hat{\theta} = \frac{3}{0.148184} = 20.25$$

$\Rightarrow$ The mode $= \theta/2 = 10.125$ #

*pb 11.32 p.242

Policies have a deductible of 100. Seven losses are observed, with values 120, 180, 200, 270, 300, 1000, and 2500. Ground-up losses have a Pareto distribution with $\theta = 400$ and α unknown. Determine the maximum likelihood estimate of α.

Ans:

$$L(\alpha) = \prod_{j=1}^{7} \frac{f(x_j|\alpha)}{1 - F(100|\alpha)} = \frac{\prod_{j=1}^{7} \frac{\alpha\, 400^{\alpha}}{(400+x_j)^{\alpha+1}}}{\left[\left(\frac{400}{400+100}\right)^{\alpha}\right]^7}$$

$$\therefore \ell(\alpha) = 7\ln\alpha + 7\alpha\ln 400 - (\alpha+1)\sum_{j=1}^{7}\ln(400+x_j) - 7\alpha\ln 0.8$$

$$\therefore \ell(\alpha) = 7\ln\alpha + 7\alpha\ln 400 - (\alpha+1)(47.2888) - 7\alpha\ln 0.8$$

where $\sum_{j=1}^{7}\ln(400+x_j) \simeq 47.29$

pdf for pareto - α, θ

$$f(x) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}}$$

&

$$S(x) = \left(\frac{\theta}{x+\theta}\right)^{\alpha}$$

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$\therefore\ \ell(\alpha) = 7\ln\alpha - 3.79\alpha - 47.29$

$\therefore\ \ell'(\alpha) = 7\alpha^{-1} - 3.79$

to get $\hat{\alpha}$, Set $\ell'(\alpha) = 0 \Rightarrow \hat{\alpha} = \dfrac{7}{3.79} = 1.847$

* pb 11.27 p. 241 Text book

Determine $\hat{\theta}^*$, the maximum likelihood estimate of θ_p restricted by Sylvia's

Ans: claim that $\boxed{\theta_s = 2\theta_p}$

For exp. distn $\bar{x}_p = \hat{\theta}_p = 1,000$ and $\bar{x}_s = \hat{\theta}_s = 1500$

(the two means for both samples)

The likelihood with the given restriction is (using i to index observations from Phil's bulbs and j to index observations from Sylvia's bulbs)

$$L(\theta^*) = \prod_{i=1}^{20}(\theta^*)^{-1}\exp(-x_i/\theta^*)\prod_{j=1}^{10}(2\theta^*)^{-1}\exp(-x_j/2\theta^*)$$

$$\propto (\theta^*)^{-30}\exp\left[-\sum_{i=1}^{20}\frac{x_i}{\theta^*} - \sum_{j=1}^{10}\frac{x_j}{2\theta^*}\right]$$

$$= (\theta^*)^{-30}\exp\left(\frac{-20\bar{x}_p}{\theta^*} - \frac{10\bar{x}_s}{2\theta^*}\right)$$

$$= (\theta^*)^{-30}\exp\left(\frac{-20(1000)}{\theta^*} - \frac{10(1500)}{2\theta^*}\right)$$

$$= (\theta^*)^{-30}\exp\left(\frac{-20,000}{\theta^*} - \frac{7,500}{\theta^*}\right) = (\theta^*)^{-30}\exp\left(\frac{-27,500}{\theta^*}\right)$$

$$\Rightarrow \ell(\theta^*) = -30\ln\theta^* - 27,500/\theta^*$$

to get $\hat{\theta}^*$, set $\ell'(\theta^*) = 0$

$$\Rightarrow -30(\theta^*)^{-1} + 27,500(\theta^*)^{-2} = 0 \qquad \times(\theta^*)^2$$

$$\therefore\ \theta^* = \frac{27,500}{30} = 916.67$$

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