Lecture ②

Defn ④: Survival fn (Reliability fn) (دالة البقاء، (دالة الموثوقية))

The survival fn is defined by

$$S(x) = S_X(x) = pr(X > x)$$
$$= 1 - F_X(x)$$

props

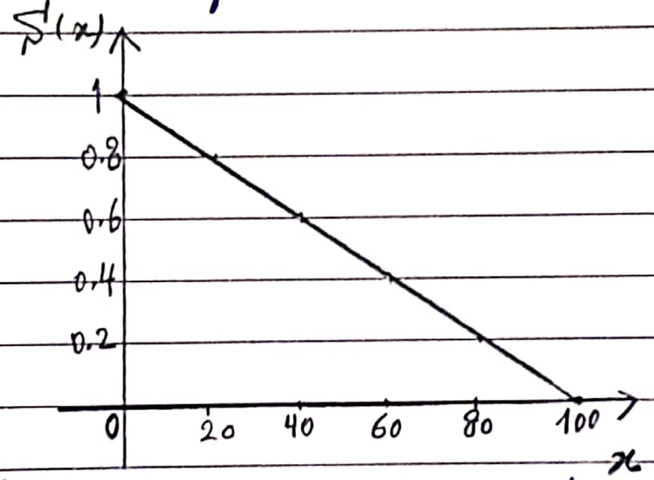
① $0 \leq S(x) \leq 1 \quad \forall x$

② $S(x)$ is non increasing Continuous fn

③ $\lim_{x \to -\infty} S(x) = 1$ and $\lim_{x \to \infty} S(x) = 0$

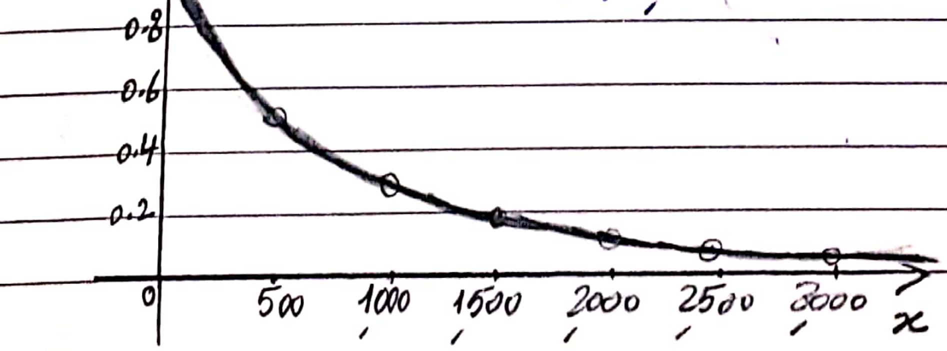
$\Rightarrow$ For Model ①: The age at death of a randomly selected birth.

$$S_1(x) = 1 - 0.01x, \qquad 0 \leq x < 100$$



, Model ②: The # of dollars paid on a randomly selected automobile bodily injury claim

$$S_2(x) = \left(\frac{2,000}{x + 2,000}\right)^3, \qquad x \geq 0$$



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Model ③: The # of automobile bodily injury claims in one Year from a randomly selected insured automobile

$$S_3(x) = \begin{cases} 0.5, & 0 \leq x < 1 \\ 0.25 & 1 \leq x < 2 \\ 0.13 & 2 \leq x < 3 \\ 0.05 & 3 \leq x < 4 \\ 0 & x \geq 4 \end{cases}$$

Model ④: the total dollars in medical malpractice claims paid in one year owing to events at a randomly selected hospital.

$$S_4(x) = 0.3\, e^{-0.00001x}, \quad x \geq 0$$

Defn ⑤

For continuous r.v X, the probability density fn (density fn) is defined as:

$$f(x) = f_X(x) = F'(x) = -S'(x)$$

i.e it's the derivative of the distn fn (negative the derivative of survival fn)

Defn ⑥

For discrete r.v X, the probability fn (probability mass fn) is defined as: $p(x) = p_X(x) = pr(X = x)$

Note ① If $F(x)$ is continuous at x then $pr(X=x)=0$; otherwise the probability is the size of the jump.

② $pr(a < X \leq b) = F(b) - F(a)$

$$= S(a) - S(b)$$

③ For continuous r.v X, $pr(a \le X \le b) = \int_a^b f(x)\,dx$

$F(b) = \int_{-\infty}^{b} f(x)\,dx$, and $S(b) = \int_b^{\infty} f(x)\,dx$

④ For discrete r.v X, $F(x) = \sum_{y \le x} p(y)$ and

$S(x) = \sum_{y > x} p(y)$

• For our four Models:

Model ① $f_1(x) = 0.01, \quad 0 < x < 100$

Model ② $f_2(x) = \dfrac{3(2{,}000)^3}{(x+2{,}000)^4}, \quad x > 0$

Model ③

$$p_3(x) = \begin{cases} 0.50, & x=0 \\ 0.25, & x=1 \\ 0.12, & x=2 \\ 0.08, & x=3 \\ 0.05, & x=4 \end{cases}$$

, clearly $\sum_x p(x) = 1$

and Model ④: the distn in Model ④ is mixed so we can write the prob. density fn

$$f_4(x) = \begin{cases} 0.7 \\ 0.000003\, e^{-0.00001x}, & x > 0 \end{cases}$$

In this case, we have 0.7 discrete probability at $x=0$ and only 0.3 continuous probability that distributed over the positive values where $x > 0$.