1

lecture ⑨

P.68 Textbook

Ex: Suppose X and Y are jointly distributed random Variables having the density function

$$f_{XY}(x,y) = \frac{1}{y} e^{-(x/y)-y} \quad \text{for } x, y > 0$$

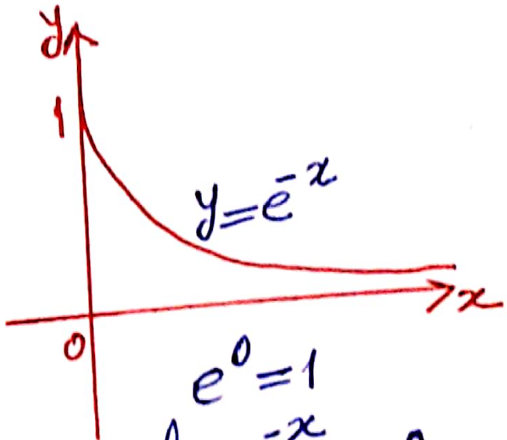
find $f_{X|Y}(x|y)$, $E[X|Y=y]$

Conditional prob. of X given $Y=y$

Conditional Expectation of X given $Y=y$

Ans:

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$



$e^0 = 1$

, $\lim_{x\to\infty} e^{-x} = 0$

$$f_Y(y) = \int_0^\infty f(x,y)\,dx$$

$$f_Y(y) = \int_0^\infty \frac{1}{y} e^{-(x/y)-y}\,dx$$

$$f_Y(y) = \frac{e^{-y}}{y} \int_0^\infty e^{-(x/y)}\,dx$$

$$f_Y(y) = \frac{e^{-y}}{y} \left[\frac{e^{-(x/y)}}{-1/y} \right]_0^\infty$$

$$\therefore f_Y(y) = \frac{e^{-y}}{\cancel{y}} [0 + \cancel{y}] = e^{-y}, \quad y > 0$$

$$\therefore f_{X|Y}(x|y) = \frac{\frac{1}{y}\, e^{-(x/y)\cancel{-y}}}{\cancel{e^{-y}}}$$

$$f_{X|Y}(x|y) = \frac{1}{y}\, e^{-(x/y)}$$

for $x, y > 0$

for $X \sim exp(\lambda)$
$f(x) = \lambda e^{-\lambda x}$, $\lambda > 0$

$$\therefore X|Y \sim exp\left(\frac{1}{y}\right)$$

$\therefore E[X|Y=y] = y$ which is the Conditional expectation of X given $Y=y$ #

Random Sums p. 57 Textbook

let $X = \xi_1 + \xi_2 + \cdots + \xi_N$ be a sum of independent and identical r.v's $\xi_1, \xi_2, \ldots, \xi_N$ where N is a discret r.v which is independent of $\xi_1, \xi_2, \ldots$ and $P_N(n) = pr\{N=n\}$ for $n = 0, 1, 2, \ldots$

Simply, we Write

$$X = \begin{cases} 0 & \text{if } N = 0 \\ \xi_1 + \xi_2 + \cdots + \xi_N & \text{if } N > 0 \end{cases}$$

3/

* The marginal prob. density fn for X is

$$f_X(x) = \sum_{n=0}^{\infty} f_{X|N}(x|n)\, P_N(n)$$

Remember

$$pr(A) = \sum_i pr(A|B_i)\, pr(B_i)$$

law of total prob.

Defn

$$E[g(X)] = \sum_{n=0}^{\infty} E[g(X) | N=n]\, P_N(n)$$

$$\therefore\ E[g(X)] = E\big[E[g(X) | N=n]\big]$$

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للدالة $g(X)$ لمّا بـ $N=n$

Note that :

$E[g(X) | N=n]$ is the conditional expectation of the function $g(X)$ given that $N=n$.

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