

Some Important Discrete Distributions

* Bernoulli distⁿ

توزيع برنولي

X ~ Ber(p), 0 ≤ p ≤ 1

Its mass prob. fn is

pr(X=x) = p^x (1-p)^(1-x), x ∈ {0, 1}

Moments: E(X) = p, Var(X) = p(1-p) = pq

q = 1 - p

* Binomial distⁿ

توزيع ذي طرئين

X ~ Bin(n, p), n ∈ N and 0 ≤ p ≤ 1

pr(X=x) = (n choose x) p^x (1-p)^(n-x)

for x = 0, 1, 2, ..., n

Moments: E(X) = np, Var(X) = np(1-p) = npq

Note: (n choose x) = n! / (x!(n-x)!)

* Poisson distn

توزيع بواسون

$X \sim \text{poisson}(\lambda), \lambda > 0$

$$\text{pr}(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, 2, \dots$$

Moments: $E(X) = \lambda$, $\text{Var}(X) = \lambda$
المتوسط التباين

* Geometric distn

$X \sim \text{Geom}(p)$

$0 < p < 1$

if its mass prob. fn is

$$\text{pr}(X=k) = p(1-p)^k, k=0, 1, 2, \dots$$

Note: X counts # of failures prior to the first success
عدد الفشل قبل النجاح الأول

$$E(X) = \frac{1-p}{p}, \text{Var}(X) = \frac{1-p}{p^2}$$

OR $\text{pr}(X=k) = p(1-p)^{k-1}, k=1, 2, \dots$

$$E(X) = \frac{1}{p}, \text{Var}(X) = \frac{1-p}{p^2}$$

pb 1.3.1 p. 24

of elements of this event (total heads is three)

$$= C_3^5 = \binom{5}{3} = \frac{5!}{3!2!} = \frac{5 \times 4 \times 3!}{3! \cdot 2 \times 1} = 10$$

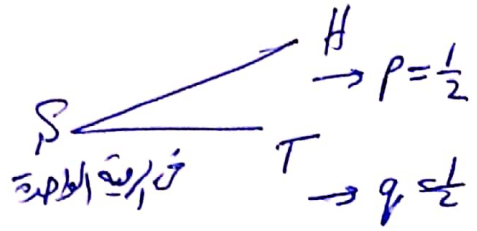
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pr (that the total of heads is three)

$$= \text{pr}(X=3)$$

$$= \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$$= 10 \left(\frac{1}{8}\right) \left(\frac{1}{4}\right) = \frac{10}{32} = \frac{5}{16}$$



OK for tossing a coin 5 times

of outcomes in the sample space is $2^5 = 32$

$$\Rightarrow \text{pr}(X=3) = \frac{10}{32} = \frac{5}{16}$$

Note: $C_3^5 = 10$
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pb 1.3.3 p.24

X Counts # of failures prior to the first success

$$\Rightarrow X \sim \text{Geom}(p)$$

$$p = 0.05$$

$$\text{pr}(X=10) = p(1-p)^{k-1}, k=1, 2, \dots$$

$$= 0.05(1-0.05)^9$$

$$= 0.05(0.95)^9$$

$$= 0.0315$$

Variance $E(X) = \frac{1}{p}$, $\text{Var}(X) = \frac{1-p}{p^2}$ تذكر!
 mean $\frac{1}{p}$ المتوسط متوسط متوسط

4 / pb 1.2.9 p. 17 Revise (Lecture 2)

EX Determine the distribution function, mean & Variance corresponding to the triangular density.

$$f(x) = \begin{cases} x & \text{for } 0 \leq x \leq 1 \\ 2-x & \text{for } 1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Ans: $F_X(x) = \int_{-\infty}^x f(t) dt$

for $0 \leq x \leq 1$

$$F_X(x) = \int_0^x t dt = \left[\frac{t^2}{2} \right]_0^x = \boxed{\frac{x^2}{2}}$$

Now, $F(1) = \frac{1}{2}$

* for $1 \leq x \leq 2$

$$F_X(x) = \frac{1}{2} + \int_1^x (2-t) dt$$

$$= \frac{1}{2} + \left[2t - \frac{t^2}{2} \right]_1^x$$

$$= \frac{1}{2} + \left[(2x - \frac{x^2}{2}) - (2 - \frac{1}{2}) \right]$$

$$F_X(x) = \frac{1}{2} + 2x - \frac{x^2}{2} - \frac{3}{2}$$

$$F_X(x) = \boxed{2x - \frac{x^2}{2} - 1}$$

Now, $F(2) = 2(2) - \frac{4}{2} - 1 = \boxed{1}$

as we expect

$$\therefore F_X(x) = \begin{cases} 0 & , x < 0 \\ \frac{x^2}{2} & , 0 \leq x \leq 1 \\ 2x - \frac{x^2}{2} - 1 & , 1 \leq x \leq 2 \\ 1 & , x > 2 \end{cases}$$

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