Some Important Discrete Distributions

* Bernoulli dist$\underline{n}$ توزيع برنولي

$$X \sim Ber(p) \quad , \quad 0 \le p \le 1$$

Its mass prob. f$\underline{n}$ is

$$pr(X = x) = p^{x}(1-p)^{1-x} \quad , \quad x \in \{0, 1\}$$

Moments: $E(X) = p$, $Var(X) = p(1-p) = pq$

, $q = 1 - p$ → النجاح

الفشل

* Binomial dist$\underline{n}$ توزيع ذي الحدين

$$X \sim Bin(n, p), \quad n \in N \text{ and } 0 \le p \le 1$$

$$pr(X = x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

for $x = 0, 1, 2, \dots, n$

Moments: $E(X) = np$, $Var(X) = np(1-p)$
$= npq$

Note: $\binom{n}{x} = \frac{n!}{x!(n-x)!}$

* 2 poisson distn   توزيع بواسون

$$X \sim poisson(\lambda), \quad \lambda > 0$$

$$pr(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots$$

Moments: $E(X) = \lambda$, $Var(X) = \lambda$

المتوسط   التباين

* Geometric distn

$X \sim Geom(p)$, $\quad 0 \leq p \leq 1$

iff its mass prob. fn is

$$pr(X=k) = p(1-p)^k, \quad k = 0, 1, 2, \ldots$$

$$E(X) = \frac{1-p}{p}, \quad Var(X) = \frac{1-p}{p^2}$$

Note: X counts # (عدد) of failures (مرات الفشل) prior to the first success قبل تحقيق أول نجاح

OR $pr(X=k) = p(1-p)^{k-1}, \quad k = 1, 2, \ldots$

$$E(X) = \frac{1}{p}, \quad Var(X) = \frac{1-p}{p^2}$$

- pb 1.3.1 p. 24

\# of elements of this event (total heads is three)

$$= C_3^5 = \binom{5}{3} = \frac{5!}{3!\,2!} = \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!} \cdot 2 \times 1} = 10$$

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pr (that the total of heads is three)

$$= pr\,(X=3)$$

$$= \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$$

$$= 10\left(\frac{1}{8}\right)\left(\frac{1}{4}\right) = \frac{10}{32} = \frac{5}{16}$$

S — رمية العملة

H $\to p=\frac{1}{2}$

T $\to q=\frac{1}{2}$

OK for tossing a coin 5 times

# of outcomes in the sample space is $2^5 = 32$

$$\Rightarrow pr(X=3) = \frac{10}{32} = \frac{5}{10}$$

Note: $C^5_3 = 10$ لأن عندي 5 رميات أختار 3

Pb 1.3.3 p.24

X counts # of failures prior to the first success

$$\Rightarrow X \sim Geom\,(P)$$

$$P = 0.05$$

$$pr(X=10) = P(1-P)^{k-1}, \quad k=1,2,\ldots$$

$$= 0.05\,(1-0.05)^9$$

$$= 0.05\,(0.95)^9$$

$$= 0.0315$$

إضافة! يمكن حساب كلاً من mean (الوسط الحسابي) 5 Variance (التباين)

$$E(X) = 1/P, \quad Var(X) = \frac{1-P}{P^2}$$

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pb 1.2.9 p. 17 (Revise Lecture ②)

EX Determine the distribution function, mean & Variance corresponding to the triangular density.

$$f(x) = \begin{cases} x & \text{for } 0 \le x \le 1 \\ 2-x & \text{for } 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Ans: $F_X(x) = \int_{-\infty}^{x} f(t)\,dt$

for $0 \le x \le 1$

$$F_X(x) = \int_0^x t\,dt$$

$$= \left[\frac{t^2}{2}\right]_0^x = \boxed{\frac{x^2}{2}}$$

Now, $F(1) = \frac{1}{2}$

* for $1 \le x \le 2$

$$F_X(x) = \frac{1}{2} + \int_1^x (2-t)\,dt$$

$$= \frac{1}{2} + \left[2t - \frac{t^2}{2}\right]_1^x$$

$$= \frac{1}{2} + \left[\left(2x - \frac{x^2}{2}\right) - \left(2 - \frac{1}{2}\right)\right]$$

$$F_X(x) = \frac{1}{2} + 2x - \frac{x^2}{2} - \frac{3}{2}$$

$$F_X(x) = \boxed{2x - \frac{x^2}{2} - 1}$$

Now, $F(2) = 2(2) - \frac{4}{2} - 1 = \boxed{1}$ as we expect

$$\therefore F_X(x) = \begin{cases} 0 & , x < 0 \\ \frac{x^2}{2} & , 0 \le x \le 1 \\ 2x - x^2/2 - 1 & , 1 \le x \le 2 \\ 1 & , x > 2 \end{cases}$$