Lecture ②

Probability Review

Ex p. 11

urn	# gold coins G	# silver coins S
I_1	4	8
I_2	3	9
I_3	6	6

Find the prob. of selecting a gold coin $pr(G)$??

Ans:

$$pr(G) = \sum_{i=1}^{3} pr(G \mid I_i)\, pr(I_i)$$

$$pr(I_1) = pr(I_2) = pr(I_3) = \frac{1}{3}$$

$$pr(G) = pr(G \mid I_1)\, pr(I_1) + pr(G \mid I_2)\, pr(I_2) + pr(G \mid I_3)\, pr(I_3)$$

$$pr(G) = \frac{4}{12}\left(\frac{1}{3}\right) + \frac{3}{12}\left(\frac{1}{3}\right) + \frac{6}{12}\left(\frac{1}{3}\right)$$

$$\therefore pr(G) = \frac{1}{9} + \frac{1}{12} + \frac{1}{6} = \frac{13}{36}$$

* Some fundamental Concepts

- We denote the random Variables by

$$X, Y, Z, \ldots$$

and $x, y, z, \ldots$ for real numbers

- The distribution f_n of the r.v X is defined as

$$F(x) = pr(X \leq x), \quad -\infty < x < \infty$$

* X is called discrete r.v if it takes distinct قيم مميزة (مختلفة) values $x_1, x_2, \ldots$ such that

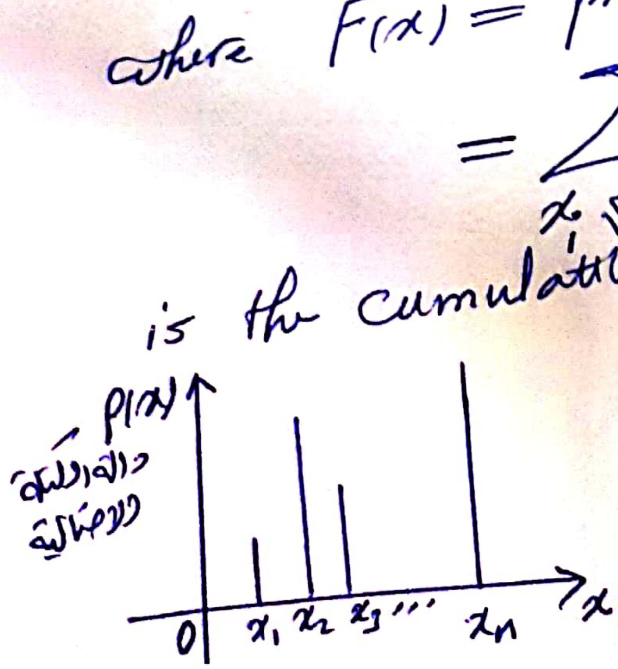
① $pr(X = x_i) = a_i > 0$ for $i = 1, 2, \ldots$

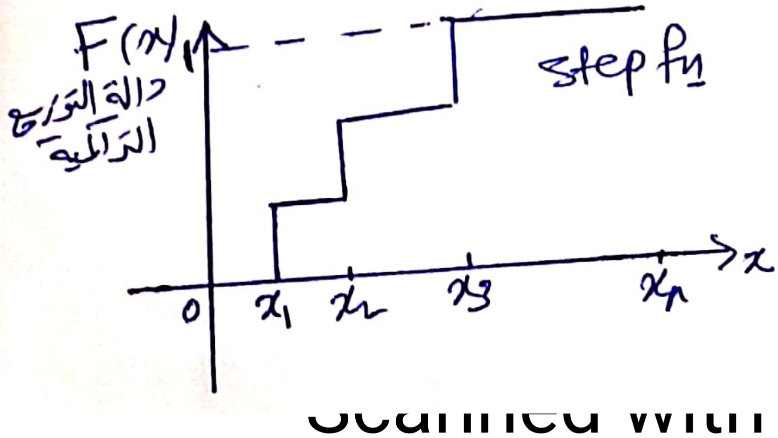
and $\sum_i a_i = 1$

② the f_n $p(x_i) = F(x_i) - F(x_{i-1})$ is called prob. mass f_n (pmf)

where
$$F(x) = pr(X \leq x) = \sum_{x_i \leq x} p(x_i)$$

is the cumulative dist$_n$ f_n (cdf)





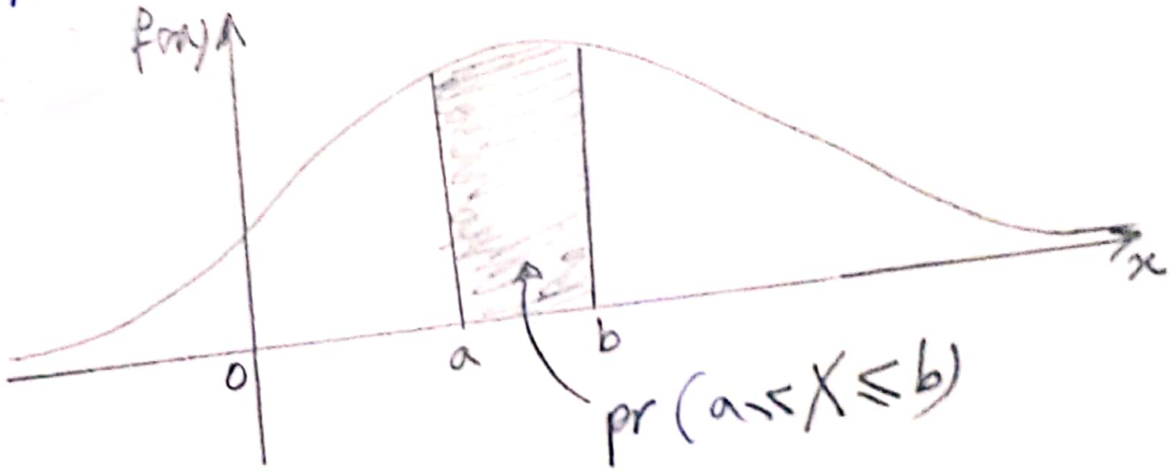
* X is called continuous r.v if it takes infinite or non-countable set of values such that

① $pr(X=x)=0$ for every value x

and $\int_{-\infty}^{\infty} f(x)\,dx = 1$

② $pr(a<X\leq b) = \int_a^b f(x)\,dx$

where $f(x)$ is called a probability density fn (pdf)



Note that:

1) the cumulative distn fn is

$$F(x) = pr(X\leq x)$$

$$= \int_{-\infty}^{x} f(\xi)\,d\xi \text{ or } \int_{-\infty}^{x} f(t)\,dt$$

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2) $f(x) = \dfrac{dF}{dx}$

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Moments and Expected values

the expected value for the f_n $g(X)$ is given by

$$E[g(X)] = \sum_i g(x_i)\, p(x_i)$$

, X is discrete r.v

$$, E[g(X)] = \int_{-\infty}^{\infty} g(x)\, f(x)\, dx$$

, X is continuous r.v

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Moments and Expected values

For discrete r.v X | **For continuous r.v X**

① $E(X) = \sum_i x_i \, p(x_i)$ | $E(X) = \int_{-\infty}^{\infty} x \, f(x) \, dx$

is the expected value of X (the mean), i.e $\mu = E(X)$.

② $E(X^r) = \sum_i x_i^r \, p(x_i)$ | $E(X^r) = \int_{-\infty}^{\infty} x^r \, f(x) \, dx$

is the r^{th} moment of X about origin.

③ Variance $V(X)$ is defined as:

$$V(X) = \sigma_X^2 = E(X-\mu)^2$$

$$= E(X^2) - \mu^2$$

$V(X) = \sum_i (x_i - \mu)^2 p(x_i)$ | $V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) \, dx$

where $\sigma_X = \sqrt{V(X)}$ is called standard deviation.
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