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Lecture 1: Stochastic processes

Textbook: "An Introduction to stochastic Modelling" - 4th Edition

Defn 1

A stochastic process is a family of random variables (r.v's) X_t or $X(t)$ where t is a parameter, $t \in T$ and T is called index set.

$T = \{0, 1, 2, \dots\}$ discrete

or

$T = [0, \infty)$ continuous

Defn 2

State space is the range of possible values for r.v's X_t

Note that: Stochastic processes are determined by their state space, index set T and the dependence relations among the r.v's X_t

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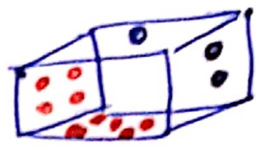
- **Examples of stochastic processes**



① Tossing a dice

$T = \{1, 2, 3, \dots, n, \dots\}$ is the index set

مرة/ مرتين

X_t may represent the number in the upper face

$\therefore$ The stochastic process is $\{X(t) : t \in T\}$

i.e $X_1, X_2, X_3, \dots$

where $S = \{1, 2, 3, 4, 5, 6\}$ is the discrete state space.

② $\{X_t\}$, where X_t represents the number of defects for a certain product in the interval $(0, t]$ is considered as a stochastic process.

③ $\{X_t\}$, where X_t indicates the number of cars in the interval $(0, t]$, along a highway is a stochastic process.

④ $\{X_n\}$, $n = 0, 1, 2, \dots$ where X_n represents the day's weather. طقس اليوم

X_0 طقس اليوم الأول
X_1 " " الثاني
X_2 " " الثالث
...

i.e $X_0, X_1, X_2, X_3, \dots$ is a stochastic process.

Note that The state space may be $S = \{1, 2\}$ $\quad 1 \to$ rainy, $2 \to$ dry

or $S = \{0, 1, 2\}$ $\quad 0 \to$ dry, $1 \to$ rainy and $2 \to$ sunny.

نعني بـ $X_0=0$ طقس جاف ، $X_0=1$ ، $X_0=2$ ، وهكذا لباقي المتغيرات العشوائية

Probability Review

1. the random Variable X (r.v X) is a Variable that takes its values by chance.
متغير يأخذ قيم بالصدفة

2. the Sample space Ω, is the Set of all outcomes of an experiment
أو تجربة

3. The event A is a subset of Ω

i.e $A \subseteq \Omega$

such that:

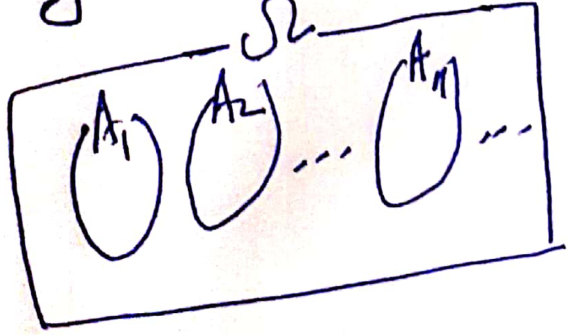
i) $0 = pr(\phi) \leq pr(A) \leq pr(\Omega) = 1$

ϕ is the impossible event
الحدث مستحيل

Ω is the sure event
الحدث المؤكد

ii) $pr\left[\bigcup_{n=1}^{\infty} A_n\right] = \sum_{n=1}^{\infty} pr(A_n)$

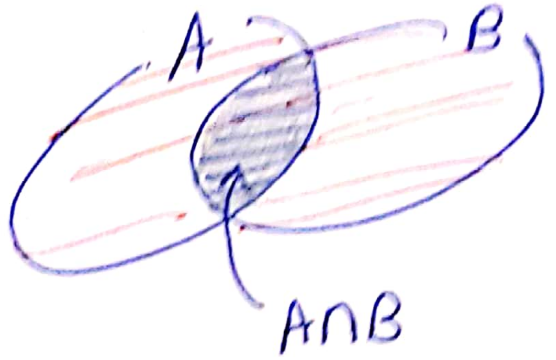
where $A_1, A_2, A_3, \ldots, A_n, \ldots$ are mutually exclusive events (disjoint events)
أحداث متنافية



$A_i \cap A_j = \phi$, $i \neq j$

Not that:

$$pr(A \cup B) = pr(A) + pr(B) - pr(A \cap B)$$

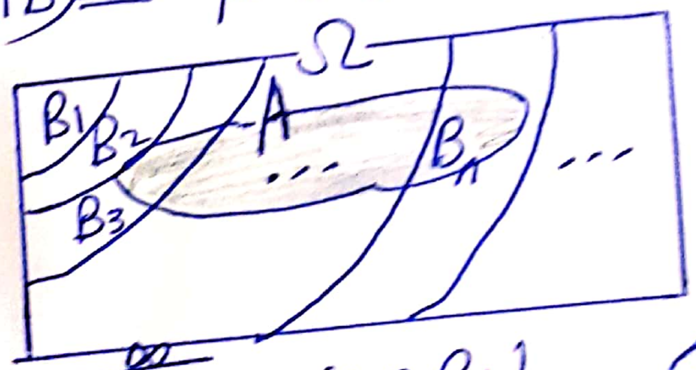


• Conditional probability & law of total probability

$$pr(A|B) = \frac{pr(A \cap B)}{pr(B)}, \quad pr(B) \neq 0$$

is the Conditional prob. of A given B ①

$$\Rightarrow pr(A \cap B) = pr(A|B)\, pr(B)$$



$$\because pr(A) = \sum_{i=1}^{\infty} pr(A \cap B_i) \quad ②$$

(from the opposite fig.)

①, ② $\Rightarrow$

$$\therefore pr(A) = \sum_{i=1}^{\infty} pr(A|B_i)\, pr(B_i)$$

i.e. $pr(A) = pr(A|B_1)\, pr(B_1) + pr(A|B_2)\, pr(B_2) + \dots$

which is called law of total probability.

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