GENERAL MATHEMATICS 2

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Main Contents

1. First-Order Linear Differential Equations

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The first-order linear differential equation has the form

$$y' + P(x)y = Q(x) \ ,$$

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To solve the first-order linear differential equation, we do the following steps:

1. Compute the integrating factor $\mu(x) = e^{\int P(x) \ dx}$.
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Solution: Write the differential equation in the form $y' + P(x)y = Q(x)$.

$$x\ y' + y = x^2 + 1 \Rightarrow y' + \frac{1}{x}y = \frac{x^2+1}{x}$$

Divide both sides by x

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The integrating factor is $\mu(x) = e^{\int \frac{1}{x}\ dx} = e^{\ln|x|} = x$

Remember: $\int \frac{1}{x}\ dx = \ln|x| + c$ and $e^{\ln f(x)} = f(x)$

$$\text{The general solution : } \ y(x) = \frac{1}{x}\int x\left(\frac{x^2+1}{x}\right)\ dx = \frac{1}{x}\int (x^2+1)\ dx$$

$$\Rightarrow y(x) = \frac{1}{x}\left(\frac{x^3}{3} + x\right) + c\ .$$

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(2) The particular solution:

$$y(1) = 0 \Rightarrow (1^2)(e^1 + c) = 0 \Rightarrow e + c = 0 \Rightarrow c = -e$$

The particular solution is $y(x) = x^2(e^x - e)$.